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EQUILIBRIUM PRICE DISPERSION

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It is shown that equilibria with dispersed prices exist in environments with identical and rational agents on both sides of the market. In particular, the original Stigler model of nonsequential search often has many equilibria, some with price dispersion. Also, price dispersion holds in equilibrium in general if search is "noisy," i.e., there is some chance of learning two or more prices when an agent is looking for one price.

1. INTRODUCTION

THE PURPOSE OF THIS STUDY is to define and characterize equilibrium in a market for a consumer good in which firms set their own prices and price information is costly to consumers. Particular attention will be paid to the conditions required for an equilibrium to involve price dispersion, i.e., the state where some firms charge different prices than others.

An important paper by Stigler [**16**] has stimulated much research on the optimal search strategy of a consumer when faced with a nondegenerate distribution of prices for a good (see, for example, [**6** or **9**]). The reason why different firms charge different prices is not discussed in this literature. Nevertheless, there is a considerable amount of empirical evidence that even in a market for a seemingly homogeneous good, price dispersion is not an uncommon phenomenon.[2] In the present study results from this consumer search literature will be used in specifying the demand side of a market for a consumer good where firms set their own prices.

Several previous studies have also succeeded in building models with dispersed price equilibria. However, the models examined here show that equilibrium price dispersion with rational agents may occur in simpler environments than previously thought. For example, many models contain some form of *ex ante* heterogeneity: in Reinganum [**11**] firms have different costs of production; in Salop and Stiglitz [**15**], consumers have different search costs; and in Wilde and Schwartz [**18**], consumers have different propensities to search with such propensities being independent of the economic value of search, i.e., some people love to shop, while others avoid it at all costs. In contrast, we confine our attention to models where firms' costs are identical, consumers are identical, and consumers search only to lower the expected costs of acquiring a desired commodity, balancing the monetary cost of search against its monetary benefit. Nor does one need a continual stream of poorly informed agents to support equilibrium dispersion. Following most of the literature we only examine rational expectations equilibria, i.e., in equilibrium firms know consumer search behavior, and consumers know the true distribution of prices. Therefore, the existence of

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[2] Pratt, Wise, and Zeckhauser [**10**] and Stigler [**16**] provide evidence on this point.

equilibrium price dispersion in these models demonstrates that *ex ante* heterogeneity in neither costs, tastes, nor rationality is necessary to explain sustained variation in prices.

What appears to be crucial for equilibrium price dispersion is an *ex post* heterogeneity in consumer information. This is what also drives many of the previous models: the stochastic advertising mechanism in Butters **[3]** and the stochastic nonsequential search in Wilde **[17]**, both force an ex post (i.e., after consumers have all their information) heterogeneity in how much various consumers know. If consumers differ sufficiently in search costs (as in Salop and Stiglitz **[15]**) or in their fixed propensities to search (as in Wilde and Schwartz **[18]**), they will purchase different amounts of information, leading again to *ex post* heterogeneity of information. We shall see that firms will offer differing prices precisely when there is a positive, but not certain, probability that a randomly observed consumer knows only one price. It is then clear that any mechanism which forces this will generate dispersion in prices. What is surprising is that, as in Section 3.2, the *ex post* heterogeneity may still occur when there is no *a priori* reason to expect it.

In the present study a framework is developed in which several types of consumer search can be analyzed. Two general search methods are then considered: nonsequential search and what we call noisy sequential search. To the authors' knowledge these two methods encompass all the nonsystematic search strategies presented in the literature to date.[3] When one of these search methods is specified, a market equilibrium with that type of search can be defined where firms maximize their expected profit, given their (correct) beliefs about consumer behavior, and consumers minimize their expected cost of purchasing one unit each given their (correct) beliefs about the distribution of prices in the market.[4]

In Section 2, the framework used in this study is developed. In Section 3, it will be shown that when nonsequential and noisy sequential search are considered, a dispersed price equilibrium can exist. With the nonsequential search method a consumer must decide how many price quotations to observe before any are received. Although most research has concentrated on cases where nonsequential search is inferior to sequential search, it is straightforward to construct environments where this is not the case. For example, suppose a consumer wants to purchase one unit of a good, but cannot visit any firm. Instead, the consumer has to write to a firm to learn the price it is charging. It takes a week for a firm to reply and another week to deliver the good if the consumer wants to purchase. A sequential search strategy is one in which the consumer writes to one firm and then waits for a reply before deciding whether to write for another price quotation or purchase at the lowest price observed. The nonsequential search problem is to determine how many letters to send (at the same time), given a

[3] A search method is termed nonsystematic if any price observed by a consumer can be envisaged as a random draw distribution of prices in the market.

[4] At any equilibrium considered in the present study the agents will be assumed to know a great deal. How agents acquired this information is not discussed.

fixed cost per letter. If the consumer must purchase in two weeks a nonsequential search strategy will be preferred to a sequential search strategy.[5]

In Section 3.2 it is shown that a dispersed price equilibrium can exist with nonsequential search, even when all consumers face the same cost of search. To the authors' knowledge this result is new.

In Section 3.3 noisy sequential search is considered. In this case, if a consumer pays to guarantee that one price quotation is observed, there is a known probability more than one price will be observed. An example will help explain this method. Suppose a consumer wants to buy one unit of a good and can guarantee observing one price by purchasing a newspaper.[6] There is, however, a known probability the newspaper will contain two firms' price quotations. In this case the strategy of the consumer which minimizes the expected cost of purchasing is to buy a newspaper, look at the price offered, and then either purchase at the lowest price observed, or buy another newspaper and observe more prices. It is shown that a dispersed price equilibrium can exist with noisy search.

2. THE FRAMEWORK

Throughout the study the following framework will be used. Suppose there is a large number of firms that produce and sell a particular good and a large fixed number of consumers who buy it. Let μ denote the (finite) measure of consumers per firm.[7] Assume that firms face the same production costs; specifically, suppose any firm's marginal cost is constant and equal to r. Each firm selects the price it will charge, and thus different firms may offer different prices. Let F indicate the distribution function describing the prices charged by firms in the market, i.e., $F(p)$ indicates the proportion of firms that charge a price no greater than p, for any p.

At any equilibrium to be considered it will be assumed that consumers know the distribution of prices, F, or at least act as if they know it. However, consumers are assumed not to know which firm is charging which price. To obtain information about this, a consumer searches a subset of the set of firms (in ways specified below) to obtain price quotations. Any price quotation received by a consumer is envisaged as a random draw from the distribution F. A consumer can only purchase from a firm whose price quotation he has received. If a consumer has chosen to purchase from a firm charging p, one unit will be bought if p is no greater than some fixed p^*; otherwise no amount of the

[5] In general, the optimal policy will be to choose a number of letters to be sent and to accept the lowest reply if it is less than the chosen reservation price. If the consumer's time preference is great enough, the optimal strategy will be a purely nonsequential strategy.

[6] The firm's name and address is also presented in the newspaper so the consumer will know where to purchase.

[7] Specifically, we are thinking of the following process. Let N be an integer. Suppose that there are N firms and μN^2 consumers, each demanding $1/N$ units of the good. Suppose that the consumers are allocated at random and independently among the firms. Then as $N \to \infty$, the total demand per firm becomes deterministic and equal to μ.

good will be bought. It will be assumed throughout that $p^* \geq r$ for no market will exist if $p^* < r$. There is a well-known difficulty with this "box" demand function: since there is no consumer surplus if p is the price paid, then no individual will ever want to participate in the market if he has to pay search costs to find a firm which surely charges p^*. Since some of our equilibria are degenerate at p^*, as was the case in Diamond **[4]**, this would indicate that the market would fail to exist. This is not an essential difficulty. If instead p^* was the monopoly price of a downward sloping demand curve, the demand being unity for any $p \leq p^*$, then there would be some consumer surplus, which is assumed sufficient to cover the costs of deciding to participate, and the analysis below would remain valid since no firm would ever find it optimal to charge a price in excess of the monopoly price.

In general, a firm's expected profit will depend on (a) the price it charges, (b) the prices other firms charge, and (c) the search method used by consumers. It should lead to no confusion if we simplify notation and let $\Pi(p)$ denote a firm's expected profit when it charges p and (b) and (c) are well specified. Note that no expected profit maximizing firm will choose a price greater than p^* or less than r. Hence, without loss of generality, it will be assumed throughout that for any distribution function F considered $F(p^*) = 1$, and $F(r - \epsilon) = 0$ if $\epsilon > 0$.

The method of search used by consumers is crucial in determining the expected profit of a firm, and thus important in analyzing a market equilibrium. In the next section two different search methods are considered. The principal objective is to characterize the market equilibria obtained, when firms maximize their expected profits and consumers minimize their expected cost of purchasing, given the search method used.

3. NONSEQUENTIAL AND NOISY SEARCH

In this section market equilibria with two different methods of search are analyzed. With both of these search methods a consumer may observe more than one price quotation before deciding whether to buy or search again. In these cases a firm not only has to worry about the reservation price used by a customer but also the other prices he/she will observe. For example, a firm may be offering a price less than the maximum acceptable price of a consumer who receives a price quotation, but still not sell to that consumer as an even lower price was observed. As the problems faced by a firm are essentially the same with either method of search discussed in this section, the firm side of the market in both cases will be presented first.

3.1. *The Firms*

The most important information for a firm to have is the distribution of prices of other firms and the consumers' search strategies. In our models we assume that the consumers use the following strategy: observe n prices and then purchase at the lowest price observed if and only if that price is no greater than $\tilde{p}$, the reservation price. If all n prices are greater than $\tilde{p}$, the consumer will observe more prices. Thus, consumer search behavior may be summarized by the pair $(\langle q_n\rangle_{n=1}^{\infty}, \tilde{p})$, where q_n denotes the probability a randomly selected consumer observes n prices before comparing the lowest price observed with $\tilde{p}$, the reservation price. Note that while $\tilde{p}$ is the same for all consumers, it is allowed that some consumers may observe more prices than others before comparing them with $\tilde{p}$. The generality of this characterization will be clear when consumer search behavior is outlined. Without loss of generality, assume $\tilde{p} \geq r$.

DEFINITION 1: Given $(\langle q_n\rangle_{n=1}^{\infty}, \tilde{p})$, a *firm equilibrium* is a pair $(F(\cdot), \Pi)$, where $F(\cdot)$ is a distribution function and Π is scalar, such that (a) $\Pi = \Pi(p)$, for all p in support of $F(\cdot)$ and (b) $\Pi \geq \Pi(p)$, for all p.

Condition (a) implies that all firms earn the same expected profits at a firm equilibrium, whereas (b) implies there is no incentive for any firm to change its price. Note that at a firm equilibrium $(F(\cdot), \Pi)$, $F(\tilde{p}) = 1$. The following lemma allows us to concentrate on specific types of firm equilibria.

LEMMA 1: *If $(\langle q_n\rangle_{n=1}^{\infty}, \tilde{p})$ is such that $q_1 \neq 1$ and $(F(\cdot), \Pi)$ is an associated firm equilibrium, $F(\cdot)$ is either continuous with connected support, or concentrated at r.*

PROOF: Suppose $F(\cdot)$ has a discontinuity at some p', where $r < p' \leq \tilde{p}$, i.e., $F(p'+) > F(p'-)$. If $q_1 \neq 1$, there is a strictly positive probability that a consumer who observes the price of a firm charging p' will also search another firm charging p'. Hence, if this firm lowered its price infinitesimally it would increase its expected profits, as the negligible decline in the profit per sale would be more than offset by the increase in that firm's expected total sales. This implies that if $F(\cdot)$ has a discontinuity at some point $p' > r$, it cannot be part of a firm equilibrium, if $q_1 \neq 1$.

Assume $F(\cdot)$ is constant on some region $[p_1, p_2]$, $p_1 < p_2$, in the convex hull of its support. In this case a firm charging $p_1 > r$ can raise its price to $p_1 + \epsilon < p_2$ $(\epsilon > 0)$ and lose none of its purchasing customers. If $p_1 = r$ and $F(r) < 1$, then raising its price from r will not cause a firm to lose all customers and will raise the profit per sale from zero to a positive amount. Hence, the expected profit of a firm charging p_1 is less than one charging $p_1 + \epsilon$, violating the equal profit equilibrium condition. This implies the support of $F(\cdot)$ must be connected, if $F(\cdot)$ is to be part of a firm equilibrium. This establishes the lemma.

Given the above lemma, the expected profit of a firm charging p can be written as

$$(1) \qquad \Pi(p) = \begin{cases} (p-r)\mu \sum_{k=1}^{\infty} q_k k(1-F(p))^{k-1}, & \text{if } p \leq \tilde{p}, \\ 0, & \text{if } p > \tilde{p}.^{8} \end{cases}$$

The next lemma characterizes all possible firm equilibria.

[8]Note that $\Pi(p) = 0$ for any p, if $\tilde{p} = r$.

LEMMA 2: *Three cases exhaust the possibilities for firm equilibria*:

(i) *Given* $(\langle q_n\rangle_{n=1}^{\infty}, \tilde{p})$ *with* $q_1 = 1$, *the unique firm equilibrium* $(F(\cdot), \Pi)$ *is the monopoly price equilibrium*,

$$\Pi = \mu(p^* - r), \quad \text{and} \quad F(p) = \begin{cases} 0, & \text{if } p < p^*, \\ 1, & \text{if } p \geq p^*. \end{cases}$$

(ii) *Given* $(\langle q_n\rangle_{n=1}^{\infty}, \tilde{p})$ *with* $q_1 = 0$, *the unique firm equilibrium* $(F(\cdot), \Pi)$ *is the competitive price equilibrium*,

$$\Pi = 0, \quad \text{and} \quad F(p) = \begin{cases} 0, & \text{if } p < r, \\ 1, & \text{if } p \geq r. \end{cases}$$

(iii) *Given* $(\langle q_n\rangle_{n=1}^{\infty}, \tilde{p})$ *with* $0 < q_1 < 1$ *and* $\tilde{p} > r$, *the unique firm equilibrium* $(F(\cdot), \Pi)$ *is such that* $F(\cdot)$ *is continuous with compact support* $[\underline{p}, \tilde{p}]$, $\tilde{p} > \underline{p} > r$, *and*

$$\Pi = \mu q_1(\tilde{p} - r) = \mu(\underline{p} - r)\sum_{n=1}^{\infty} nq_n > 0,$$

defines Π *and* $\underline{p}$. *If* $0 < q_1 < 1$ *and* $\tilde{p} = r$, *the unique market equilibrium is where all firms charge the competitive price*, r, *and* $\Pi = 0$.

PROOF: Claim (i) of the lemma follows immediately from (1). To establish claim (ii) suppose $(\langle q_n\rangle_{n=1}^{\infty}, \tilde{p})$ is such that $q_1 = 0$. From Lemma 1 we know that at any firm equilibrium $(F(\cdot), \Pi)$, $F(\cdot)$ is either concentrated at r or continuous and strictly increasing on the convex hull of its support. First, it is clear that if $q_1 = 0$ the competitive price is a firm equilibrium, since any firm that raises its price above r will lose all its purchasing customers. Second, it is claimed that there is no other firm equilibrium. Suppose there is another equilibrium. That firm equilibrium $(F(\cdot), \Pi)$ will be such that $F(\cdot)$ is continuous with compact support. Let $\bar{p} = \sup_{F(p)<1} p$. As $p \to \bar{p}$, $F(p) \to 1$, and since $q_1 = 0$

$$\Pi(p) \to (\bar{p} - r)\,\mu \sum_{k=1}^{\infty} kq_k(1 - 1)^{k-1} = 0.$$

But $\Pi(p) = \Pi$ for all p in the support of $F(\cdot)$, and the support is an interval. Hence, $\Pi = 0$. However, at any p where $0 < F(p) < 1$,

$$\Pi(p) = (p - r)\,\mu \sum_{k=1}^{\infty} nq_k(1 - F(p))^{k-1} > 0,$$

which contradicts the assumption that $\Pi(p)$ is constant. This establishes claim (ii).

Suppose $(\langle q_n\rangle_{n=1}^{\infty}, \tilde{p})$ is such that $0 < q_1 < 1$. It follows that if $\tilde{p} > r$, no firm equilibrium can have all firms charging r as any firm could raise its price infinitesimally, keep some customers, and make more money. Thus, if $\tilde{p} > r$ and

$F(\cdot)$ is part of a firm equilibrium, $F(\cdot)$ must be continuous with compact support. Hence, at any firm equilibrium when $0 < q_1 < 1$ and $\tilde{p} > r$,

$$\Pi = \Pi(p) = (p - r)\,\mu \sum_{k=1}^{\infty} kq_k(1 - F(p))^{k-1}$$

for any p in support of $F(\cdot)$. This implies

$$\frac{\Pi}{(p-r)\mu} = \sum_{k=1}^{\infty} kq_k(1 - F(p))^{k-1}.$$

The right hand side of the above is a C^∞ monotone increasing function of $1 - F(p)$ and hence has a C^∞ increasing inverse $\Phi(\cdot)$. Thus,

$$F(p) = 1 - \Phi(\Pi/(p - r)\mu),$$

for any p in support of $F(\cdot)$. It is straightforward to check that $\sup_{F(p)<1} p = \tilde{p}$. Hence, $\Pi = \Pi(\tilde{p}) = (\tilde{p} - r)\mu q_1$. The equal profit condition implies

$$\Pi(\underline{p}) = (\underline{p} - r)\,\mu \sum_{k=1}^{\infty} q_k k = \Pi, \qquad \text{where} \quad \inf_{F(p)>0} p = \underline{p}.$$

This establishes the first part of claim (iii). The second part of claim (iii) follows from the fact that a firm would lose all its customers if it raised its price above r.

3.2. *Nonsequential Search*

As discussed above, suppose that the delays in communication and the time rate of preference together make nonsequential search superior to sequential search. In this case a consumer will choose the number of prices to observe before receiving any offers. Suppose the cost of receiving n price quotations is cn. Thus, if each price quotation is a random draw from the distribution $F(\cdot)$, the expected cost of purchasing when n prices are obtained is

$$cn + \int_0^\infty np(1 - F(p))^{n-1}\,dF(p).$$

Note that this is a convex function of n with a unique minimum when n is allowed to be any positive real number. Hence there exists either a unique integer n^* that minimizes the expected cost of purchasing, or there are two integers n^* and $n^* + 1$ that both minimize the expected cost of purchasing. With nonsequential search a consumer will purchase at the lowest price observed if and only if it is no greater than p^*. We shall call p^* the effective reservation price when the nonsequential search method is used.

DEFINITION 2: The triple $(F(\cdot), \Pi, \langle q_n \rangle_{n=1}^{\infty})$ is a *market equilibrium with nonsequential search* if, and only if, for fixed p^* and cost of search $c > 0$, (a)

$(F(\cdot), \Pi)$ is a firm equilibrium given $(\langle q_n \rangle_{n=1}^{\infty}, p^*)$, and (b) $\langle q_n \rangle_{n=1}^{\infty}$ is generated from the expected cost minimizing strategies of the consumers given $F(\cdot)$.

If at a market equilibrium with nonsequential search $F(\cdot)$ is such that it is concentrated at p^*, it is termed a *monopoly price market equilibrium*. Similarly, if $F(\cdot)$ is concentrated on r, it is termed a *competitive price market equilibrium*. If at equilibrium $F(\cdot)$ is not concentrated at any price, it is termed a *dispersed price equilibrium*. Note that from Lemma 2, these three are the only possible types of market equilibria.

THEOREM 1: *If $c > 0$ and if $(F(\cdot), \Pi, \langle q_n \rangle_{n=1}^{\infty})$ is a market equilibrium with nonsequential search, then it is either a monopoly price equilibrium or a dispersed price equilibrium. Furthermore, a monopoly price equilibrium always exists.*

PROOF: Suppose all firms charge r. Then all consumers would search only once. However, with this search behavior firms would raise prices to p^*. The second claim follows as consumers will search only once if all firms charge p^*. This completes the proof.

The above result establishes that a monopoly price market equilibrium always exists with nonsequential search, given the search costs faced by consumers are strictly positive. The existence of dispersed price equilibria has not been established. Below it is shown that a dispersed price equilibrium can exist, even if all consumers face the same search costs.

THEOREM 2: *Suppose all consumers face the same cost per price observation, $\bar{c} > 0$. In this case there are one, two, or three market equilibria with nonsequential search; one monopoly price equilibrium, and zero, one, or two dispersed price equilibria. Further, there exists a $c^* > 0$ such that* (i) *$\bar{c} < c^*$ implies there are two dispersed price market equilibria, and* (ii) *$\bar{c} > c^*$ implies there are no dispersed price equilibria.*

PROOF: We showed in Theorem 1 above that there exists a monopoly price market equilibrium with nonsequential search. To establish the other results claimed in Theorem 2 three claims are stated and proved.

CLAIM 1: If all consumers face the same cost of search $\bar{c} > 0$, then at any market equilibrium $(F(\cdot), \Pi, \langle q_n \rangle_{n=1}^{\infty})$, $q_1 + q_2 = 1$ and $1 \geq q_1 > 0$.

PROOF OF CLAIM 1: As all consumers face the same cost of search, they will all observe the same number of price quotations, or be indifferent to searching n or $n+1$ times for some positive integer n. If all consumers search more than once, at a firm equilibrium all firms will charge r. But then all consumers would search only once. Thus, $q_1 > 0$ and $q_1 + q_2 = 1$. This completes the proof of claim 1.

Before stating claim 2 some notation is developed. For any $q \in [0,1]$, set $q_1 = q$, $q_2 = 1 - q$, and let $(F^q(\cdot), \Pi^q)$ denote the associated firm equilibrium. Such firm equilibria are now characterized.

Claim 2: For any fixed q with $0 < q < 1$, the unique associated firm equilibrium $(F^q(\cdot), \Pi^q)$ satisfies

(a) $$\Pi^q = (p^* - r)\mu q = (p - r)\mu\left[q + 2(1-q)(1 - F^q(p))\right]$$

for any p in the support of $F^q(\cdot)$,

(b)

(2) $$F^q(p) = \begin{cases} 0, & \text{if } p < \underline{p}(q), \\ 1 - \left[\dfrac{p^* - p}{p - r}\right]\left[\dfrac{q}{2(1-q)}\right], & \text{if } \underline{p}(q) < p \leq p^*, \\ 1, & \text{if } p > p^*, \end{cases}$$

and

(c) $$\underline{p}(q) = (p^* - r)\frac{q}{(2-q)} + r.$$

Proof of Claim 2: Lemma 2 established that if $q \in (0,1)$, a firm equilibrium $(F^q(\cdot), \Pi^q)$ exists and $F^q(\cdot)$ is continuous and strictly increasing on the convex hull of its support. (a) above follows from the equal profit condition of a firm equilibrium. (b) and (c) can now be established from (a). This completes the proof of Claim 2.

Let V denote the expected difference in the purchasing price paid by a consumer who observes two prices instead of one price quotation. It follows

$$\begin{aligned} V &= \int_0^{p^*} p\,dF(p) - 2\int_0^{p^*} p(1 - F(p))\,dF(p) \\ &= \int_0^{p^*} F(p)\,dp - \int_0^{p^*} \left[F(p)\right]^2 dp \qquad \text{(by integration by parts).} \end{aligned}$$

Clearly V depends on the distribution of prices faced by the consumer. Considering only those distributions specified in (2), V is a function of q,

(3) $$V(q) = \int_{\underline{p}(q)}^{p^*} F^q(p)\,dp - \int_{\underline{p}(q)}^{p^*} \left[F^q(p)\right]^2 dp$$

for any $q \in (0,1)$. A consumer will strictly prefer to observe two prices instead of one if and only if $V(q) > \bar{c}$. Further, a consumer will be indifferent to observing one or two prices if and only if $V(q) = \bar{c}$.

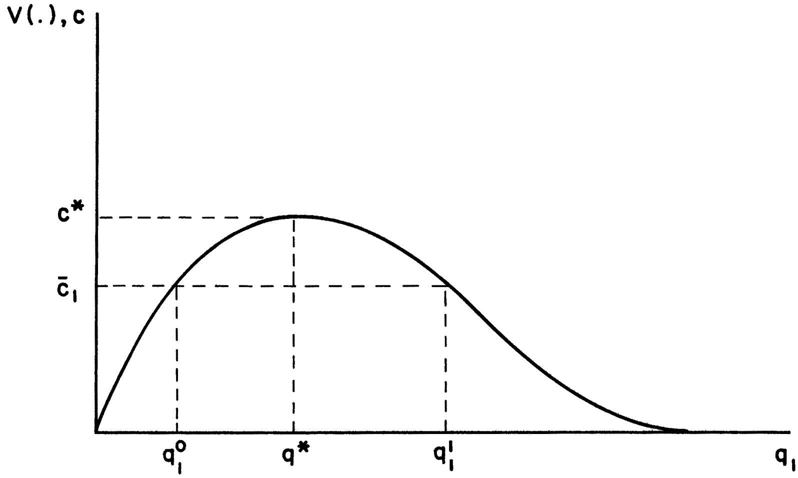

FIGURE 1.

CLAIM 3: $V(q)$ has a unique maximum at some q^*, $0 < q^* < 1$. Further, $V(\cdot)$ is strictly increasing (decreasing) if $q^* < q < 1$ (if $0 > q > 1$), and $V(q) \to 0$ as $q \to 0$ or $q \to 1$.

PROOF OF CLAIM 3: As the proof of this claim is of little or no economic interest it is relegated to an Appendix.

The proof of Theorem 3 can now be established from the above claims and inspection of Figure 1. Claims 1 and 2 demonstrate that if we are looking for a dispersed market equilibrium we need only consider distributions of the type specified in (2) with $q_1 + q_2 = 1$. Figure 1 graphs the function $V(\cdot)$. The shape of this function is as demonstrated in Claim 3. Suppose the cost of a price observation is $\bar{c}$, as shown in Figure 1. In this case, if q_1^0 or q_1^1 of the consumers obtain only one price quotation, there is a dispersed price market equilibrium with nonsequential search as $V(q_1^i) = \bar{c}$, $i = 0, 1$. If the common search cost exceeds c^*, no dispersed price market equilibrium exists, and only one exists if the common search cost is c^*. This establishes the claims made in the Theorem.

3.3. *Noisy Search*

With noisy search a consumer pays c to receive an unknown number of price quotations. The consumer can then purchase at the lowest price observed, or search again. Although the consumer does not know how many price quotations will be received from paying cost c, the probability any particular number of price quotations will be observed from any search is assumed to be known. Making a harmless change in the notation previously used, let q_k denote the probability k prices will be observed from one search, $k = 1, 2, \ldots,$ and $\sum_{k=1}^{\infty} q_k = 1$.

Throughout this section it will be assumed that all consumers face the same cost search, $c > 0$. The optimal strategy for a consumer faced with noisy search is a straightforward generalization of sequential search. The strategy that minimizes the expected cost of purchasing one unit involves the use of a reservation price, z. If z denotes the lowest price observed to date, the consumer will be indifferent between purchasing and searching again. Formally, the reservation price, z, is the price which equates the marginal cost of search to the expected benefit, i.e.,

$$c = \int_0^z (z - p)\, dJ(p), \tag{4}$$

where $J(\cdot)$ is the distribution of the lowest price observed during one search. Since $F(\cdot)$ denotes the distribution of prices,

$$J(p) = \sum_{k=1}^{\infty} q_k \big(1 - (1 - F(p))^k\big), \qquad \text{for any } p.$$

Integration by parts yields

$$c = \int_0^z J(p)\, dp = \int_0^z \sum_{k=1}^{\infty} q_k \big(1 - (1 - F(p))^k\big)\, dp. \tag{5}$$

Hence, for any given $\langle q_n \rangle_{n=1}^{\infty}$ and $F(\cdot)$, (5) can be used to generate the reservation price used by all consumers. For noisy search, the effective reservation price, $\bar{p}$, is defined by

$$\bar{p} = \min(p^*, z). \tag{6}$$

If consumers utilize the effective reservation price $\bar{p}$, at any equilibrium no firm will offer a price greater than $\bar{p}$. Any firm that did would have no purchasing customers. Consequently, at any equilibrium each consumer will search only once as all prices observed will be no greater than $\bar{p}$. Utilizing the above results implies that at any firm equilibrium the search strategy of all consumers can be characterized by $(\langle q_n \rangle_{n=1}^{\infty}, \tilde{p})$, if all consumers face the same cost of search, where $\tilde{p}$ denotes the reservation price the firms believe consumers are using. Hence, the results obtained in Section 3.1 can be used. It should be noted that with noisy search $\langle q_n \rangle_{n=1}^{\infty}$ is a parameter and is not generated endogenously as in nonsequential search.

Definition 3: For any given $\langle q_n \rangle_{n=1}^{\infty}$, and any cost of search $c > 0$, a *market equilibrium with noisy search* is a triple $(F(\cdot), \Pi, \bar{p})$ where (a) $(F(\cdot), \Pi)$ is a firm equilibrium given $(\langle q_n \rangle_{n=1}^{\infty}, \bar{p})$, and (b) $\bar{p}$ is the effective reservation price given $F(\cdot)$.

For given c the market equilibrium will depend on $\langle q_n \rangle_{n=1}^{\infty}$. Suppose $\langle q_n \rangle_{n=1}^{\infty}$ is such that $q_1 = 1$. In this case the model is identical to the sequential search model with consumers facing the same search cost. The well-known result, e.g., [4], is that the unique equilibrium is the monopoly price equilibrium. Suppose now that $\langle q_n \rangle_{n=1}^{\infty}$ is such that $q_1 = 0$. Lemma 2(ii) established that the unique firm equilibrium is a competitive price equilibrium, if $q_1 = 0$. If all firms charge r, the effective reservation price used by all consumers will be $\bar{p} = r + c$. This defines a market equilibrium when $q_1 = 0$. It should be noted that consumers need not have perfect information about prices for a market equilibrium to be a competitive price equilibrium. For example, if each consumer observes two prices, a competitive price equilibrium is the unique market equilibrium.

The situation is not so straightforward if $0 < q_1 < 1$. For any given $\langle q_n \rangle_{n=1}^{\infty}$ where $0 < q_1 < 1$, if $\tilde{p} > r$, Lemma 2(iii) established that there is a unique firm equilibrium which is a dispersed price equilibrium. To simplify the exposition let $(F(\cdot;\tilde{p}), \Pi(\tilde{p}))$ denote the unique firm equilibrium for any $r \leq \tilde{p} \leq p^*$ when $\langle q_n \rangle_{n=1}^{\infty}$ is fixed with $0 < q_1 < 1$. If a consumer is faced with distribution function $F(\cdot;\tilde{p})$, let $\bar{p}(\tilde{p})$ indicate the reservation price used by the consumer for a fixed $c > 0$. Using Definition 3, (1), and the assumption that $0 < q_1 < 1$, we may conclude that at a market equilibrium with noisy search

$$(7) \qquad q_1(\tilde{p} - r) = (p - r) \sum_{k=1}^{\infty} k q_k (1 - F(p;\tilde{p}))^{k-1}$$

for any p in the support of $F(\cdot;\tilde{p})$, and

$$(8) \qquad \bar{p}(\tilde{p}) = \tilde{p}.$$

To verify the existence of an equilibrium the two extreme positions are first considered. Suppose $\tilde{p} = r$. Then all firms will charge r and consumers will utilize an effective reservation price $\bar{p}(r) = r + c$. Hence, at $\tilde{p} = r$, $\bar{p}(\tilde{p}) > \tilde{p}$. Next consider the case where $\tilde{p} = p^*$. From (5) and (6) it follows that $\bar{p}(p^*) \leq p^*$. As $\bar{p}(\cdot)$ is clearly continuous, there exists at least one market equilibrium. The following claim establishes that a unique market equilibrium with noisy search exists for given $\langle q_n \rangle_{n=1}^{\infty}$ when $0 < q_1 < 1$; the proof is given in the Appendix.

CLAIM 4: If $\bar{p}(\tilde{p}) = \tilde{p}$ for some $\tilde{p}$, where $r < \tilde{p} < p^*$, then $\bar{p}'(\tilde{p}) < 1$.

Claim 4 is sufficient to claim uniqueness of a market equilibrium when $0 < q_1 < 1$. The results that can be obtained for the noisy search case can now be summarized.

THEOREM 4: *For fixed* $\langle q_n \rangle_{n=1}^{\infty}$ *and* $c > 0$, *with noisy search*: (a) *the unique market equilibrium is a monopoly price equilibrium if* $q_1 = 1$, (b) *the unique market equilibrium is a competitive price equilibrium if* $q_1 = 0$, *and* (c) *the unique market equilibrium is a dispersed price equilibrium if* $0 < q_1 < 1$.

Similar results to those presented in Sections 3.1 and 3.2 can be obtained if market entry by firms is allowed. Suppose firms will continue to enter until the expected profit accruing to a firm is no greater than some $K \geq 0$. If there is a dispersed price equilibrium or a monopoly price equilibrium, allowing market entry determines the long-run number of consumers per firm. If the market equilibrium is a competitive price equilibrium, there is no equilibrium consumers per firm as firms always make zero expected profit in this case.

4. CONCLUSION

We have succeeded in demonstrating the possibility of price dispersion in equilibrium with fully rational and identical agents on both sides of the market. First, this proves that price dispersion may exist independent of the heterogeneities used by other authors. Second, this shows that equilibrium price dispersion may be a durable long-run phenomenon, not arising merely due to short-run differences in cost functions or consumer rationality. These models also have the advantage of being simple, therefore, amenable to the development of extensions and further analysis. Examples of further possible work include stability analysis, which may give further information concerning the durability of equilibrium price dispersion and reduce the multiplicity of equilibria in the nonsequential model. Another interesting generalization would be the introduction of advertising in both models, thereby having information gathered by consumers and disseminated by producers.

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APPENDIX

Proof of Claim 3: To establish the claim it will be assumed, without loss of generality, $r = 0$. Doing the indicated integration in (3) yields

$$V(q) = p^*\left[\frac{q}{2(1-q)^2}\ln\left[\frac{2-q}{q}\right] - \frac{q}{1-q}\right]$$

and

$$V'(q) = p^*\left[\frac{q-3}{2-q} + \frac{1+q}{2(1-q)}\ln\left[\frac{2-q}{q}\right]\right](1-q)^{-2}.$$

Hence, if $q \in (0, 1)$, $V'(q) = 0$ if and only if $X(q) = 0$, where

$$X(q) = \frac{2(1-q)(q-3)}{(1+q)(2-q)} + \ln\left[\frac{2-q}{q}\right].$$

Thus, if $X(q)=0$ for exactly one $q \in (0,1)$, $V(q)$ has exactly one stationary point. The following facts demonstrate $X(q)=0$ only once for $q \in (0,1)$:

(a) $$X(0)=+\infty \quad \text{and} \quad X(1)=0.$$

(b) $$X'(q)=\frac{Y(q)}{Z(q)}=\frac{-4+16q-20q^2+8q^3}{q(2-q)^2(1+q)^2}.$$

(c) $$Z(q)>0, \quad \text{if} \quad q \in (0,1),$$

(d) $$Y(0)=-4, \quad Y'(0)=16, \quad Y(1/2)=0, \quad Y'(1/2)=2, \quad Y(1)=0, \quad \text{and} \quad Y'(1)=0.$$

The above facts imply $V(\cdot)$ has exactly one stationary point for $q \in (0,1)$. To see that $V(\cdot)$ is maximized at this stationary point it is sufficient to note that $V'(\cdot)$ is positive for some value of q close to 0, and negative for some value of q close to 1. This is sufficient by the uniqueness of a zero of V' and the continuity of $V''(\cdot)$.

Proof of Claim 4: Suppose $\tilde{p}$ is such that $\bar{p}(\tilde{p})=\tilde{p}$ and $\tilde{p}<p^*$. The derivative of (5) at this $\tilde{p}$ can be written as

$$0=\sum_{k=1}^{\infty} q_k \int_0^{\bar{p}(\tilde{p})} k(1-F(p;\tilde{p}))^{k-1} F_{\tilde{p}}(p;\tilde{p})\,dp+\bar{p}'(\tilde{p})\left[1-(1-F(\bar{p}(\tilde{p}),\tilde{p}))^k\right]$$

where

$$F_{\tilde{p}}(p;\tilde{p})=\frac{\partial F(p;\tilde{p})}{\partial \tilde{p}}.$$

Since $\bar{p}(\tilde{p})=\tilde{p}$ by assumption and $F(\bar{p}(\tilde{p});\tilde{p})=1$ by implication, we see that

$$\bar{p}'(\tilde{p})=-\sum_{k=1}^{\infty} q_k\left[\int_0^{\bar{p}(\tilde{p})} k(1-F(p;\tilde{p}))^{k-1} F_{\tilde{p}}(p;\tilde{p})\,dp\right].$$

Hence, if (a) $F_{\tilde{p}}(p;\bar{p}(\tilde{p}))<0$ and

(b) $$\left|\int_0^{\bar{p}(\tilde{p})} k(1-F(p;\tilde{p}))^{k-1} F_{\tilde{p}}(p;\tilde{p})\,dp\right|<1,$$

$\bar{p}'(\tilde{p})<1$. Thus if (a) and (b) hold, the claim is proved. To demonstrate (a) and (b) are true we divide (7) by $(p-r)$ for p in the support of $F(\cdot,\tilde{p})$ and differentiate with respect to this p and $\tilde{p}$ respectively, which yields

$$\frac{q_1(\tilde{p}-r)}{(p-r)^2}=RF_p(p;\tilde{p})$$

and

$$\frac{q_1}{(p-r)}=-RF_{\tilde{p}}(p;\tilde{p})$$

where

$$R=\sum_{k=1}^{\infty} kq_k(1-k)(1-F(p;\tilde{p}))^{k-2}\geq 0 \quad \text{and} \quad F_p(p;\tilde{p})=\frac{\partial F(p;\tilde{p})}{\partial p}.$$

Thus,

(A1) $$F_{\tilde{p}}(p;\tilde{p})=-\frac{q_1}{(p-r)R}\leq 0, \quad \text{and} \quad F_p(p;\tilde{p})=\frac{q_1(\tilde{p}-r)}{R(p-r)^2}\geq 0.$$

Further, using the above, $r < p < \tilde{p}$ implies

$$|F_{\tilde{p}}(p;\, \tilde{p})| < |F_p(p;\, \tilde{p})|.$$

It then follows

$$\text{(A2)} \qquad \left|\int_0^{\tilde{p}} k(1 - F(p;\, \tilde{p}))^{k-1} F_{\tilde{p}}(p;\, \tilde{p})\, dp\right| < \int_0^{\tilde{p}} k(1 - F(p;\, \hat{p})^{k-1} |F_p(p;\, \tilde{p})|\, dp$$

$$< \int_0^{\tilde{p}} d\left[1 - (1 - F(p;\, \tilde{p}))^k\right] = 1.$$

(A1) and (A2) imply $\bar{p}'(\tilde{p}) < 1$, if $\bar{p}(\tilde{p}) = \tilde{p} < p^*$ and the claim is proved.

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