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Source: *The Review of Economic Studies*, Apr., 1974, Vol. 41, No. 2 (Apr., 1974), pp. 219-255

Published by: Oxford University Press

Stable URL: https://www.jstor.org/stable/2296714

# Incentives and Risk Sharing in Sharecropping [1,2]

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At least from the time of Ricardo, economists have begun their investigations of how competitive markets work, how wages, rents and prices are determined, by a detailed examination of agriculture. Even today, agriculture is taken as the paradigm—and perhaps almost the only important example—of a truly competitive market (or at least this was the case until the widespread government intervention in this market). For a number of years I have been concerned with how competitive markets handle risk taking, and how risk affects real resource allocation. Risks in agriculture are clearly tremendously important, yet remarkably the traditional theoretical literature has avoided explicit treatment [3] of risk sharing in agricultural environments. The consequences of this are important. First, it makes suspect the traditional conclusions regarding sharecropping. Is it really true that sharecropping results in too low a supply of labour, because workers equate their share of output times the (value of the) marginal productivity of labour to the marginal disutility of work, whereas Pareto optimality requires the (value of the) marginal productivity of labour be equal to the marginal disutility of work? Or is it true, as Wicksell asserted, that there is no distincion between landlords hiring labour or labour renting land? Second, it leaves unanswered many of the important economic questions. How is the equilibrium share determined? Why have some economies (in the past or at present) used one distribution system, other economies used others?

Our object is to formulate a simple general equilibrium model of a competitive agricultural economy. (Other general equilibrium models of competitive economies with uncertainty have been formulated by Arrow [2] and Debreu [9], Diamond [10], and Stiglitz [14]. Each of these has its serious limitations in describing the workings of the modern capitalist economy. (See Stiglitz [15]).) The model is of interest not only for extending our understanding of these simple economies but also in gaining some insight into the far more complex phenomena of shareholding in modern corporations. Our focus is on the risk sharing and incentive properties of alternative distribution systems.

The analysis is divided into two parts. In the first, the amount of labour (effort) supplied by an individual is given, and the analysis focuses on the risk sharing aspects of sharecropping. Among the major qualitative propositions are the following.

(*a*) If workers and landlords can both " mix contracts " (i.e. workers can work for several different landlords and landlords hire workers on several different " contracts ") then the economy is productively efficient (the land-labour ratio is the same on every plot of land); if not, the economy may not be productively efficient.

[1] *First version received June* 1972; *final version received April* 1973 (*Eds.*).

[2] The research described in this paper was supported under grants from the National Science Foundation and from the Ford Foundation. The author is indebted to G. Heal, D. Newbery and G. Hughes for helpful comments.

[3] With a few exceptions. See, in particular, the important work of Cheung [6] and the extensive bibliography he provides. His conclusions closely parallel those reached by this study. His analytic approach is, however, markedly different. Since his study, two further important papers have appeared: that of Rao, which provides empirical support of some of the hypotheses advanced here, and that of Bardhan and Srinivasan, which does not explicitly treat uncertainty.

Because of Cheung's excellent discussion of it, I shall omit any further references to the literature.

(*b*) In the former case, there is, in our model, a linear relationship between the fixed payment a worker receives (his " wages ") and his share. We can thus identify a " price of risk absorption ". We show that whenever there is a pure sharecropping contract, and workers and landlords can mix contracts, the pure sharecropping contract can be dispensed with, in the sense that all the risk-sharing opportunities could be provided by combining pure wage and pure rental contracts.

(*c*) The mean marginal product of a labourer is greater or less than his mean income as the worker pays a rent or receives a wage (in addition to some share in the total output). Thus the landlord's income can be thought of as a payment for " rent of land " plus a payment for absorbing some of the labourers' proportionate share of risk. In the case of pure sharecropping (i.e. no fixed payments to or from workers), the mean marginal product is equal to the mean income.

(*d*) There will be (in our model) a pure wage (rental) system if and only if all landlords (workers) are risk neutral.

(*e*) If workers cannot mix contracts, more risk averse landlords may have a smaller number of workers per acre than less risk averse landlords (so the economy may not be productively efficient). If there are systematic relations between size of farm (wealth of landlord) and the landlord's degree of risk aversion, as one might expect, then there would be a systematic relation between size of farm and output per acre.

The derivation of further qualitative properties requires the assumptions of a " representative " landlord and a " representative " worker.

(*f*) The worker pays some fixed rent to the landlord (in addition to a share of the return) or receives a fixed wage (in addition to a " share " of the return) as the worker is less or more risk averse than the landlord. An increase in the variance of the output of the farm increases (decreases) the share of the crop paid to the landlord as well as the share of mean income received on average by the landlord if the landlord is less (more) risk averse than the worker. Other qualitative propositions are derived but have no simple interpretation.

In the second part of the paper, the supply of labour (effort) is assumed to be variable. If effort can easily (costlessly) be observed, and quantified, then the level of effort is specified in the contract. It is shown then that, contrary to the classical proposition, there is not an undersupply of labour (effort) as a result of a sharecropping system.

On the other hand, if effort (labour supply) cannot be easily observed, then sharecropping has an important *positive* incentive effect. If the landlord were risk neutral, and if there were no incentive effects (as in the models of Part I), he would absorb all the risk. Here, on the contrary, the worker still receives a share of the output. The more responsive the individual is to incentive effects the greater the incentive share (and the greater the risk he must absorb). On the other hand, whether the worker receives more or less on average than his mean marginal product depends solely on whether the elasticity of substitution between land and labour is greater or less than unity.

Although we have not developed a normative framework within which to evaluate the competitive " incentive " system we are able to establish the following: (*a*) in comparing a wage system with a sharecropping system, there is no presumption in a general equilibrium model that sharecropping reduces effort (labour) from what it would have been under a wage system with enforceable contracts; (*b*) the economy will not be " productively efficient "; there will be differences in output per acre arising because of differences in the efficacy of incentives among different individuals.

The presence of a third factor (capital) considerably complicates the analysis; we are able to show, however, that in general the capital will be entirely provided either by the landlord or the worker; in the former case, we argue that there is a greater return to closer supervision, and because of the non-convexity associated with supervision, a greater likelihood of using a wage system. This part of the paper closes with some speculative remarks about alternative incentive schemes.

## PART I. RISK SHARING WITH INELASTIC LABOUR SUPPLY

### 1. *The Basic Model*

The economy consists of two groups of individuals, landlords (who own the land, but do no work) and workers, who own no land.

On each farm, output, $Q$, is a stochastic constant returns to scale function of land, $T$, and labour, $L$:

$$Q = g(\theta)F(L, T), \qquad \ldots(1.1)$$

where $\theta$ is the " state of nature ". (1.1) has two strong implications: (*a*) " risk " is independent of inputs, e.g. rainfall affects the crop output the same way, regardless of what techniques are used to generate that output; (*b*) the returns to different farms (labourers) are perfectly correlated. Much of the literature on the economics of uncertainty has focused on the role of diversification. Risk sharing can, however, be treated as an important economic phenomenon quite apart from risk diversification; the particular assumption that we have made is chosen with that in mind.[1]

Because $F$ is homogeneous of degree one, we have

$$Q/T = g(\theta)F(L/T, 1) \equiv g(\theta)f(l), \qquad \ldots(1.2)$$

where

$$l = L/T \quad \text{and} \quad Eg(\theta) \equiv 1$$

$$\sigma_g^2 \equiv E(g-1)^2 > 0.$$

We assume, moreover, that $f$ is an increasing, concave function of $l$

$$f' > 0, \quad f'' < 0. \qquad \ldots(1.3)$$

Our primary concern in this section is the determination of the equilibrium distribution of income. We limit ourselves to linear distribution systems, i.e. if $Y_w$ is the income of a worker and $Y_r$ the income of the landlords (rentiers) then

$$Y_w = \frac{\alpha Q}{L} + \beta \qquad \ldots(1.4a)$$

$$Y_r = (1-\alpha)Q - \beta L, \qquad \ldots(1.4b)$$

where

$$0 \leqq \alpha \leqq 1.$$

Three special cases should be noted

- $\beta = 0$ the pure sharecropping system
- $\alpha = 0$ the wage system: landlords hire labour at a fixed fee
- $\alpha = 1$ workers rent land at a fixed fee.

There is no reason to expect on *a priori* grounds that the economy will be in one of these polar cases. Note that if $\beta < 0$, the worker pays the landlord a fixed fee for the use of the land, and the landlord is entitled, in addition, to a given percentage of the crop. $\beta > 0$ is the case where the worker receives a basic wage plus " incentive pay ".

[1] Since some of the variations in output are due to events which affect particular individuals (a particular worker becoming sick at harvest time) or particular pieces of land (a small creek floods, destroying the crop) there are advantages to be had from risk diversification by the landlord, in hiring more individuals, and for the worker, by working on several different plots of land. The implications of risk diversification are discussed briefly in Part II, Section 3.

Our problem then is the competitive determination of $\alpha$ and $\beta$, which determine the distribution of income as well as the distribution of risk taking.

The process for the determination of the equilibrium levels of $\alpha$ and $\beta$ is, in some ways, fundamentally different from that discussed in the usual competitive models. For there we have a single price, say the wage, for allocating a single factor, labour, and another price, rent, for allocating the other factor, land. Here we have two " prices ", " a fixed fee " and " a share " for allocating land and labour, *and risk*, and they are intertwined in a most complex way.

Moreover, the fixed fee and the share do not by themselves determine the value of a contract, since if $\alpha > 0$ the worker must know how much land he will be allowed to work, and if $\alpha < 1$, the landlord must know how much " labour " his labourers will supply. In the usual competitive analysis, *physical data*, such as the amount of land the worker is to work, plays no role in the decision making of the individual; only the price data of the wage he will receive per hour is of relevance.

In the corresponding competitive model without uncertainty there is, in effect, only one equilibrium condition: at the announced wage, all firms hire workers up to the point where the wage, $w$, is just equal to the value of the marginal product. This generates a demand curve for labour, $L^d$, and equilibrium requires demand equal supply.

Here, we must have the demand for labourers under *each* kind of contract (each specification of $\alpha$ and $\beta$) equal to the supply of labourers for that contract. Most of our analysis focuses on the contracts which will actually be made. For a contract to be signed, three conditions must be satisfied.

*Equilibrium condition* (*a*). Workers choice among existing contracts. Of the set of contracts available in the economy, there exists none which the individual worker prefers to the one which he has.[1]

*Equilibrium condition* (*b*). Landlords choice among existing contracts. Of the set of contracts available in the economy, there exists no subset which the landlord prefers to the subset which he employs.[2]

The implicit assumption underlying these two equilibrium conditions is that there is a reasonable amount of " mobility " of agricultural labourers. For traditional agricultural environments, this is probably not a very good assumption. On the other hand, in such societies, variations, for instance, in attitudes towards risk may not be as important, so that essentially a " uniform " contract develops for all workers. In that case, equilibrium conditions (*a*) and (*b*) are of no concern. Moreover, in most such societies today workers have a choice between working for a wage and working on their farm, with the resulting " risk " in the income stream. Different individuals allocate their time between these alternative " contracts " differently.

*Equilibrium condition* (*c*). Determination of available contracts. Of the set of utility equivalent contracts—the set of contracts which give the worker the same level of (expected) utility—the contract(s) signed must be the most preferred by the landlord.[3]

We explicitly assume here that the kinds of contracts offered are not determined by tradition, but are determined by economic forces.

[1] For simplicity, we have expressed the condition for the case when each worker signs a contract with only one landlord. When he can sign several contracts, i.e. allocates different fractions of his labour to different landlords with different contracts, we obtain:

*Equilibrium Condition* (*a'*). Of the set of contracts available in the economy, there exists no (feasible) subset which the worker prefers to the subset which he has.

The conditions in which equilibrium condition (*a'*) is more " reasonable " than (*a*) are discussed below.

[2] Again, if the landlord had to sign the same contract with all his workers, we would have:

*Equilibrium Condition* (*b'*). Of the set of available contracts, there exists none which the landlord prefers to the one which he employs.

[3] Given the other contracts he has signed. Provided both landlord and workers can mix contracts, the analysis could have been conducted completely symmetrically, in terms of workers hiring land. The formal symmetry breaks down later.

The remainder of this section is devoted to the elucidation of these conditions and their implications.

### 1.1. *Choice of Preferred Contract from Given Set of Contracts*

All individuals are assumed to be expected utility maximizers. We shall denote the utility function of workers by $U_w$ and of landlords (rentiers) by $U_r$. Workers are risk averse ($U''_w < 0$). In Part I we assume that landlords are risk averse ($U''_r < 0$), and that the supply of labour by a worker is inelastic. Each worker has one " unit " of labour.

Let us first look at the problem from the point of view of the workers. Income in any state of nature from a contract which has share $\alpha$, fixed payment $\beta$ per worker, and assigns to the worker an amount of land such that the labour-land ratio is $l$, is given by

$$Y_w(\theta) = \alpha g(\theta)\frac{f(l)}{l} + \beta \qquad \text{...(1.5)}$$

$$= g(\theta)x + \beta,$$

where

$$x \equiv \frac{\alpha f(l)}{l}. \qquad \text{...(1.6)}$$

(1.5) has the important implication that the income stream can be characterized by two parameters $x$ and $\beta$, alone.[1] It is easy to see that for all $\theta$, $Y_w$ increases with $x$ and $\beta$. Thus all individuals prefer, at any given $\beta$, more " $x$ " to less " $x$ " and at any given $x$ more " $\beta$ " to less " $\beta$ ". Denote the function giving the maximum value of $\beta$ attainable for any given value of $x$ in the set of available contracts by

$$\beta = \beta(x). \qquad \text{...(1.7)}$$

Then the worker maximizes

$$\max_{\{x\}} EU_w[Y_w(\theta)] = EU_w[xg(\theta) + \beta(x)]$$

or, if $\beta$ is differentiable,

$$\frac{EU'_w g}{EU'_w} = -\beta'. \qquad \text{...(1.8)}$$

Diagrammatically, we can see the nature of the solution if we write

$$EU_w[Y_w(\theta)] = V_w(x, \beta). \qquad \text{...(1.9)}$$

Since $U$ is a concave function of $Y$, and $Y$ a linear function of $x$ and $\beta$, $V(x, \beta)$ is a concave function, the indifference curves of which look as in Figure (1.1*a*). The set of possible contracts is also drawn in Figure (1.1*a*). The nature of this set is one of the objects of this study. Note that if the function $\beta(x)$ is convex and if the worker can supply fractions of his labour to different landlords, then he can attain any point on the line defined by $\max_{\{\gamma_i\}} \Sigma\gamma_i\beta(x_i)$ s.t. $\Sigma\gamma_i = 1$, $\gamma_i \geqq 0$ as in Figure (1.1*b*).

Thus, if the worker is allowed to work for more than one landlord, the contracts along the line ABC will never be signed by risk averse workers. Only the contracts A and C will be observed.

A perfectly symmetric analysis applies to the landowner. His income, $Y_r(\theta)$, if he signs a contract with share $\alpha$, fixed payment $\beta$, and assigns $l$ workers to a unit of land is,

$$Y_r(\theta) = [(1-\alpha)g(\theta)f(l) - \beta l]T, \qquad \text{...(1.10)}$$

[1] That is, the worker is indifferent among contracts with the same value of $\beta$ and $x$; there can be a higher value of $l$ if at the same time $\alpha$ is increased to keep $x$ constant.

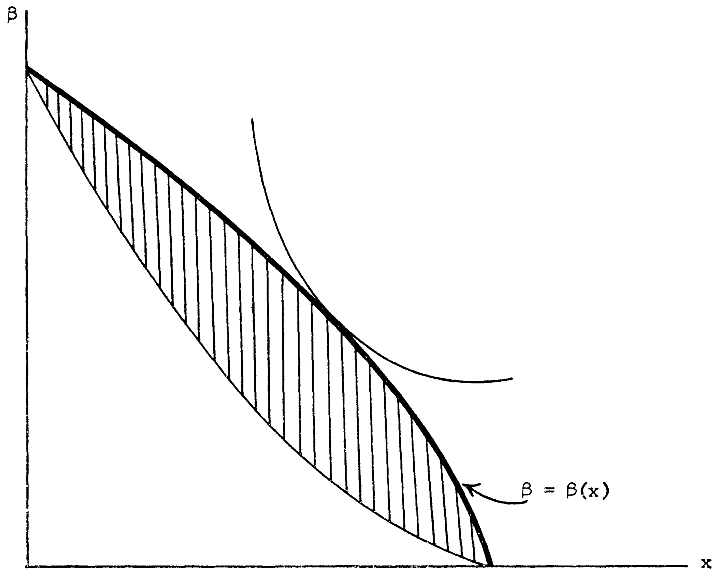


FIGURE 1.1*a*

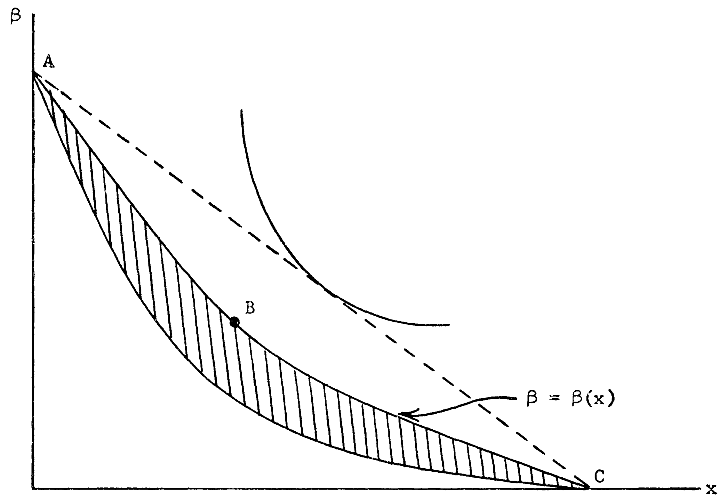


FIGURE 1.1*b*

where $T$ is the total land ownership of the landlord. This may be rewritten

$$Y_r(\theta) = \left[\left(\frac{f(l)}{l} - x\right) g(\theta) - \beta\right] lT. \qquad \text{...(1.10')}$$

His income depends not only on $\beta$ and $x$, but also on $l$.[1] The landlord must choose not only a contract, $(\beta, x)$, but also the number of such contracts (labourers per acre). Let $\hat{\beta}(x)$ = the minimum value of $\beta$ associated with any $x$. He wishes to maximize

$$\max_{\{l,\, x\}} EU_r[Y_r(\theta)] = EU_r\left[\left(\left(\frac{f(l)}{l} - x\right) g - \hat{\beta}(x)\right) lT\right]. \qquad \text{...(1.11)}$$

1 Although this makes the analysis appear slightly asymmetrical, note that we could have defined an alternative (but somewhat less natural) set of variables symmetric to $x$ and $\beta$ for the landlord. ($l$ obviously is important to the worker, but its effects are subsumed in " $x$ ".)

Again, if $\hat{\beta}$ is differentiable, he obtains as first order conditions [1]

$$\frac{EU_r'g}{EU_r'} = -\beta' \qquad \text{...(1.12a)}$$

$$EU_r'f - lf'(l)\,g = EU_r'Y_r(\theta). \qquad \text{...(1.12b)}$$

It is not so easy to depict diagrammatically the solution to (1.11), for there are three variables involved. But let us assume that we have somehow chosen $l$. Then we can write

$$EU_r[Y_r(\theta)] = V_r(x, \beta;\ l),$$

where $V_r$ is now a *convex* function of $x$ and $\beta$. Since utility decreases with $x$ and $\beta$, the only contracts which will be signed are those which, at any value of $x$, minimize $\beta$. Since workers sign only contracts which, at any value of $x$, maximize $\beta$, the set of contracts actually signed must lie along a line, i.e. $\beta(x) \equiv \hat{\beta}(x)$. Moreover, by exactly the same argument as before, if landlords can mix the kinds of contracts they sign, if $\beta(x)$ is a convex function, such as we have depicted in Figure (1.2*b*), they can attain any point on the line AC. Hence the only contracts they will sign are A and C.

**Proposition 1.** *The set of contracts actually observed must lie along a line defined by $\beta = \beta(x)$. If labour and landlords can both mix contracts, the set of contracts that are observed must lie along a straight line, i.e.*

$$\beta = -ax + b. \qquad \text{...(1.13)}$$

*If workers can mix contracts but landlords cannot, $\beta$ is a concave function of $x$; if landlords mix contracts but workers do not, $\beta$ is a convex function of $x$.*

Proposition 1 has the following immediate

**Corollary.** *There is a linear relationship in the contracts signed between the mean and standard deviation (of workers' income).*

Defining

$$\mu_w \equiv EY_w = x + \beta$$
$$\sigma_{Y_w} = x\sigma_g, \qquad \text{...(1.14)}$$

(1.13) can be rewritten

$$\mu_w = p\sigma_{Y_w} + b, \qquad \text{...(1.15)}$$

where

$$p = \frac{1-a}{\sigma_g}.$$

Note that nowhere in the analysis have we made the usual assumptions required for mean variance analysis to be applicable. $p$ is like a price of " risk " and indeed we shall show later that if workers and landlords treat it as such, the economy will possess the usual optimality properties associated with the price system.

The assumption that landlords can sign different kinds of contracts with different labourers seems a reasonable one; the other assumption required for the validity of these results, that workers can work half their time with one landlord, half with another, is somewhat more suspect, particularly in more traditional agricultural environments. The difficulty is that there is nothing in the assumptions made so far which would seem to warrant a requirement that a worker work with only one landlord. (That is, under the assumptions of constant returns to scale, there is no difference between a worker selling

[1] We shall return later to a detailed interpretation of these first-order conditions.

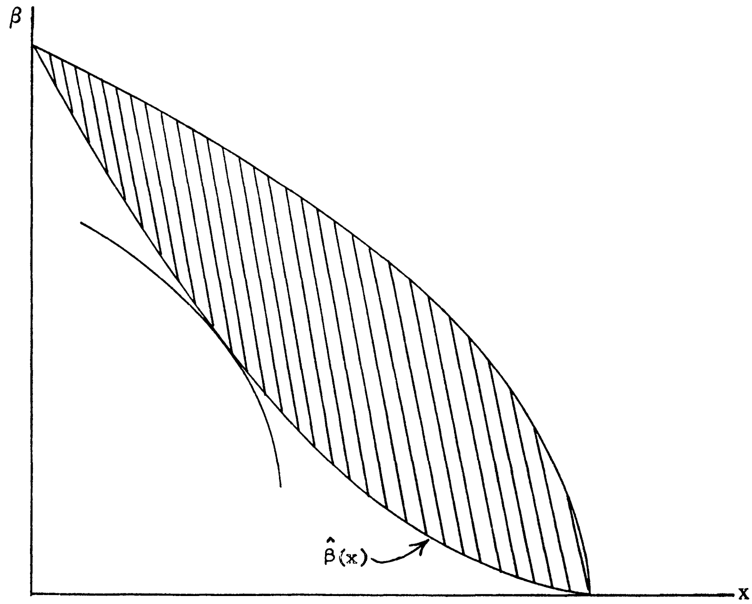


FIGURE 1.2*a*

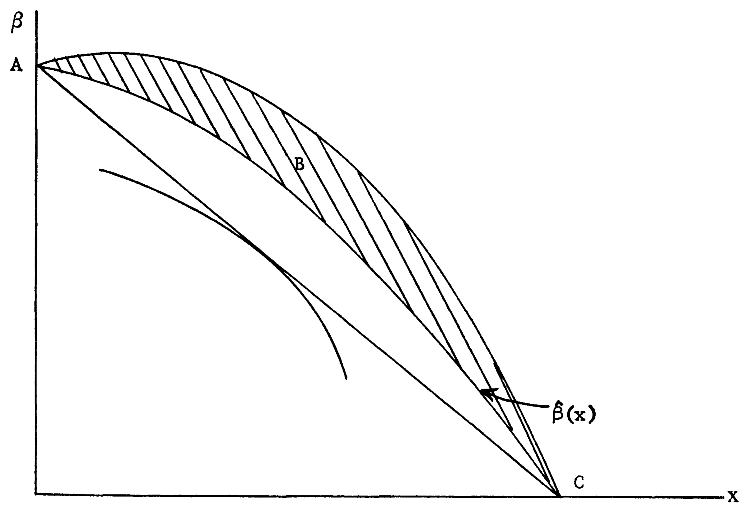


FIGURE 1.2*b*

half his time to a landlord to work on a half an acre, or selling all his time to a landlord to work on a whole acre.) The fixed costs of moving from one landlord to another, the difficulty of making sure that the labourer is really spending half his time with each of the two landlords, the suspicion that the worker will allocate more of his "effort" at certain crucial times to the contract with the greater incentive payoffs (greater $x$) [1]—all important considerations which we have omitted from our analysis—provide some explanation of why workers work only with one landlord.

[1] See below, Part II, Section 4.

1.2. *Determination of Set of Available Contracts*: *Identical Individuals*

We now turn to an examination of equilibrium condition (*c*), for a determination of the set of contracts that will be available in the economy.[1] The problem is seen most clearly in the case of the economy in which all landlords are identical and in which all workers are identical. In equilibrium, there will be only one contract observed. How are the competitive values of $\alpha$ and $\beta$ determined simultaneously?

To answer this question, we introduce the concept of " utility equivalent contracts "—contracts which yield the same level of expected utility, $U^j_w$, to the $j$th worker:

$$EU^j \equiv W^j_w = EU^j_w\left[\alpha\frac{f(l)}{l}g(\theta)+\beta\right] = EU^j_w[xg+\beta]. \qquad ...(1.16)$$

For a given value of $W_w$ we can solve (1.16) for $\beta$ as a function of $x$

$$\beta = h^j(x;\ W_w), \qquad ...(1.17)$$

with [2]

$$-\frac{\partial h}{\partial x} = \frac{EU'_w g}{EU'_w} \lesseqgtr 1 \text{ as } U''_w \lesseqgtr 0 \qquad ...(1.17a)$$

$$-\frac{\partial^2 h}{\partial x^2} = \geqq 0 \quad \text{(by Schwartz's inequality)} \qquad ...(1.17b)$$

and

$$\frac{\partial h}{\partial W_w} = \frac{1}{EU'_w} > 0. \qquad ...(1.17c)$$

The difference between $h^j$ and the function $\beta(x)$ introduced earlier should be clear. The latter is simply a description of the set of contracts available on the market; the former is a kind of " offer curve " of the individual. Indeed (1.17) is just an analytic representation of the $(x, \beta)$ indifference curves given in Figure (1.1). While we argued that under certain conditions the former had to be linear, the latter is always a concave function. The offer curve will, of course, differ from individual to individual and for each individual, it will depend on his level of utility.

Assume that the landlord knew this function, i.e. individuals tradeoffs between " $x$ " and fixed payments. Then, he would choose that particular contract which maximized his expected utility, i.e. choosing our units so the representative landlord has one unit of land, the landlord

$$\text{maximizes} \quad EU_r[(1-\alpha)f(l)g-\beta l]$$
$$= EU_r\left[\left(\left(\frac{f(l)}{l}-x\right)g-\beta\right)l\right] \qquad ...(1.18)$$

subject to the constraint that he be able to obtain workers. If $W$ is the level of utility that the representative worker can obtain elsewhere, he can obtain workers provided he offers them a contract yielding a level of expected utility at least equal to $W$.[3] Thus, we replace

[1] The problem is somewhat analogous to the problem that arises in ordinary competitive analysis when fewer commodities can be produced than the total number of commodities; given the commodities which are produced, the theory provides an explanation of the quantities that are produced and the prices they sell at. But how are the commodities that are actually produced determined?

[2]
$$EU'_w(g-1) = E[U'_w - U'_w(x+\beta)][g-1] + U'_w E(g-1)$$
$$= E[U'_w - U'_w(x+\beta)][g-1] < 0 \text{ if } U''_w < 0.$$

Where there is no ambiguity, we omit the superscript $j$ and subscript $w$.

[3] Clearly, he will not offer them a higher level of utility since that will not be maximizing his own utility. Thus he must offer them exactly $W$.

the simple price taking assumption of the usual competitive model by a "utility taking" assumption; the landlord maximizes (1.18) subject to (1.17) where he takes $W_w$ as given. The landlord makes two decisions—a choice of contract $(x, \beta)$ and a "density" decision, $l$.

The first order conditions are

$$\frac{\partial EU_r}{\partial x} = -EU'_r\left(g + \frac{\partial h}{\partial x}\right)l = 0 \qquad \text{...(1.19}a\text{)}$$

$$\frac{\partial EU_r}{\partial l} = EU'_r[(f' - x)g - \beta] = 0. \qquad \text{...(1.19}b\text{)}$$

These can be rewritten as (using (1.17$a$)

$$f' = \frac{-1}{(d \ln \beta)/dx)_U} + \frac{\alpha f}{l} = \bar{Y}_w - h\left(1 + \frac{1}{h'}\right) \qquad \text{...(1.20}a\text{)}$$

and

$$\frac{EU'_r g}{EU'_r} = \frac{EU'_w g}{EU'_w}. \qquad \text{...(1.20}b\text{)}$$

*A mean-variance interpretation.* The interpretation of the first-order conditions may be somewhat clearer in terms of mean-variance analysis. The mean and standard deviation of the landlord's income are given by

$$\bar{Y}_r \equiv (1-\alpha)f - lh$$
$$\sigma_{Y_r} \equiv ((1-\alpha)f)\sigma_g, \qquad \text{...(1.21)}$$

and (using (1.17$a$) and (1.15))

$$\frac{\partial \bar{Y}_r}{\partial \alpha} = -f(1+h') \leqq 0 \qquad \frac{\partial \bar{Y}_r}{\partial l} = f' - \bar{Y}_w + \frac{\alpha}{l}(1+h')(f - f'l)$$

$$\frac{\partial \sigma_{Y_r}}{\partial \alpha} = -f\sigma_g < 0 \qquad \frac{\partial \sigma_{Y_r}}{\partial l} = (1-\alpha)f'\sigma_g > 0.$$

If workers are risk neutral, $1 + h' = 0$, hence increasing $\alpha$ leaves the mean unchanged but reduces the variance. Thus, $\alpha$ is set at unity, and $l$ is chosen to maximize $\bar{Y}_r$ (since $\sigma_{Y_r}$ is identically zero), i.e. $f' = \bar{Y}_w$, workers receive on average their mean marginal product. Similarly, if landlords are risk neutral, they choose $\alpha$ to maximize $\bar{Y}_r$, so $\alpha = 0$, and again, $l$ is chosen so that $f' = \bar{Y}_w$. More generally, however, landlords will stop short of maximizing expected output, because as expected output increases, so does standard deviation. If $\alpha$ is given, the choice of $l$ is depicted in Figure (1.3). Increasing $l$ increases $\sigma_{Y_r}$, and, up to a point, increases $\bar{Y}_r$ as well.

If $l$ is given, the mean and standard deviation of total output is given: the standard deviation of the workers' income plus the standard deviation of the capitalists' income equals the total standard deviation,[1] and the mean income of the workers plus the mean income of the landlords equals the total mean income; thus we can represent the set of possible allocations by means of a standard Edgeworth box. This is done in Figures (1.4$a$) and (1.4$b$). Our only question is how is mean and standard deviation divided between the two groups? To the landlord, the utility level of the workers is given, and so he faces the following possibilities: he can give his workers a wage of $w = h(0;\ W)$ and absorb all the risk himself, or by increasing the share, he can increase the mean he gives the workers but decrease the standard deviation along the indifference curve WW.

[1] Throughout this and the next section, we assume that the both groups have no other sources of income.

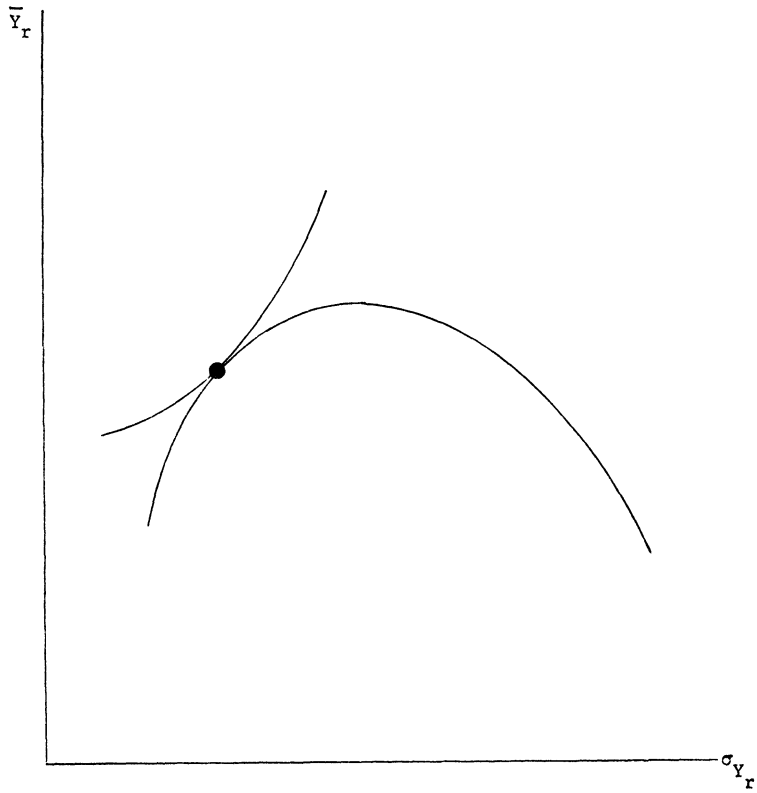


FIGURE 1.3
Choice of $l$ given $\alpha$

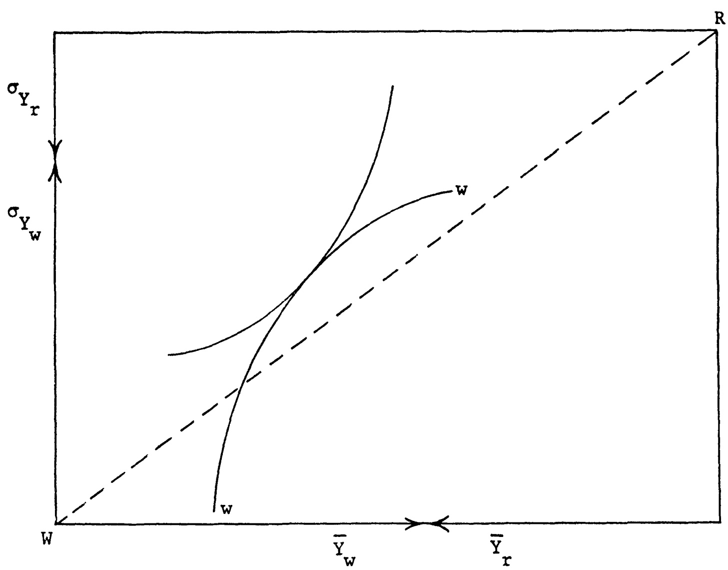


FIGURE 1.4*a*
$\beta < 0$

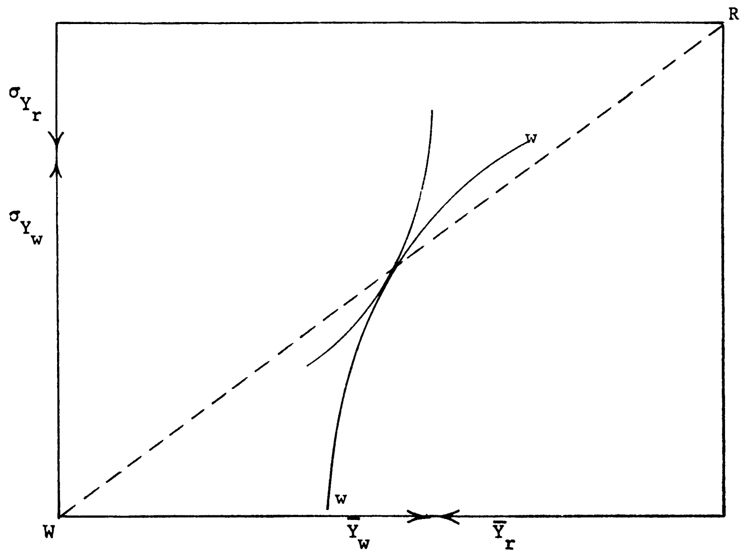


FIGURE 1.4*b*
Pure sharecropping. Choice of $\{\alpha, \beta\}$ (given $l$)

Obviously, the landlord chooses a point where his indifference curve is tangent to that of the workers. If this point of tangency occurs along the diagonal (Figure (1.4*b*)), it means that there is a pure sharecropping system, i.e. mean and standard deviation are shared proportionately; if we have a corner solution along the lower horizontal axis, it means that we have a pure wage system, and if along the upper horizontal axis, it means we have a pure rental system. Let us say that one individual is more risk averse than another if, at any value of the coefficient of variation, the increment in mean income required to compensate him for an increment in risk (standard deviation of income) is greater. (This definition clearly parallels the definition, in conventional production theory, of one sector being more capital intensive than another. Then it immediately follow that the equilibrium contract will be above, on, or below the diagonal in Figure (1.4), as the landlord is less risk averse than, has the same risk aversion as, or is more risk averse than the representative worker (that is, $\beta$ is greater than, equal to, or less than zero as landlords are less, equally, or more risk averse than workers).

*Further implications of first-order conditions.* To see in more detail the nature of the equilibrium, first we note that if $U_w''<0$, $U_r''<0$, $EU_r'g/EU_r' \leqq 1$ as $\alpha \leqq 1$, and

$$EU_w'g/EU_w' \leqq 1$$

as $\alpha \geqq 0$, so that if both groups are risk averse, we will never have a pure wage or rental system. We have thus established

**Proposition 2.** *A pure wage or pure rental system will occur if and only if workers or landlords are risk neutral. The risk neutral group absorbs all the risk, and the whole economy acts as if there were no uncertainty with factors on average receiving their mean marginal products.*

The existence of pure rental and wage systems requires introducing other factors not yet taken into account in the model.

Secondly, from (1.17*a*), we have $-h'<1$ unless [1] $U_w''=0$ or $\alpha=0$ or $g(\theta)$ is identically equal to 1. Hence from (1.20*a*) we obtain

[1] From the previous paragraph, we know that $\alpha \neq 0$ unless $U_r'' \equiv 0$,

**Proposition 3.** *If both workers and landlords are risk averse, then if workers receive a share plus a fixed fee contract, they are hired to the point where their mean marginal product is less than their average income, and by symmetry if landlords receive a share plus a fixed fee, then labourers are hired to a point where their mean marginal product is greater than their average income. The amount* $-h(1+1/h')$ *is like a " risk premium ".*

The difficult question is the determination of whether $\beta \gtrless 0$, i.e. whether labourers take more or less than their proportionate share of the risk. The answer clearly depends on the degrees of risk aversion and incomes of the two groups; there are no general theorems to be had. What we can do, however, is to ascertain, on the basis of reasonable values of the " parameters " the likely sign of $\beta$, and to determine how $\beta$ changes with certain changes in the economy.

We assume both groups have constant elasticity utility functions,

$$U_w = \frac{Y^{-\eta_w}}{\eta_w}, \quad U_r = \frac{Y^{-\eta_r}}{\eta_r},$$

where $\eta_w+1$ and $\eta_r+1$ are the Arrow-Pratt measures of relative risk aversion.

We can establish

$$\beta \gtrless 0 \quad \text{as} \quad \eta_w \lessgtr \eta_r.$$

**Proposition 4.** *The group which is relatively risk averse assumes less than its proportion of the risk.* The proof is presented in the appendix.

*Determination of $\alpha$ and $\beta$ for small variances.* Explicit expressions for the equilibrium values of the relevant variables can be derived if we assume the variance of $g$ is small. We can then make the following approximations

$$1-\frac{EU'_w g}{EU'_w} \approx A_w x \sigma_g^2 = \eta_w \frac{x}{\bar{Y}_w} \sigma_g^2$$

$$1-\frac{EU'_r g}{EU'_r} \approx A_r(f-lx)\sigma_g^2 = \eta_r \frac{(f-lx)}{\bar{Y}_r} \sigma_g^2,$$

where $A = -U''/U'$, $\eta = (U''/U')Y$ are the measures of absolute and relative risk aversion. Thus from our equilibrium conditions (1.20*b*) we obtain (after some manipulation)

$$\beta = \frac{\frac{f}{l}\left(\frac{1}{\eta_r}-\frac{1}{\eta_w}\right)}{\frac{1}{\alpha\eta_w}+\frac{1}{(1-\alpha)}\frac{1}{\eta_r}} \qquad \text{...(1.22)}$$

which may be alternatively written as

$$\alpha = \frac{A_r}{\frac{A_w}{l}+A_r} = \frac{A_r Y_r}{A_w \frac{Y_w Y_r}{l Y_w}+A_r Y_r} = \frac{\eta_r S_w}{\eta_w S_r + S_w \eta_r} = \frac{\eta_r/S_r}{\frac{\eta_w}{S_w}+\frac{\eta_r}{S_r}} \qquad \text{...(1.23)}$$

where $S_r$ = landlord's share of (mean) national income and $S_w$ = workers' share of (mean) national income. From (1.20*a*) we obtain

$$(\gamma - S_w) = \frac{(\gamma\eta_w S_r - (1-\gamma)\eta_r S_w)\eta_w\eta_r\sigma_g^2}{(\eta_w S_r + \eta_r S_w)^2} \qquad \text{...(1.24)}$$

(where $\gamma = f'l/f$) from which it is clear that shares approach " average marginal productivity shares " as $\sigma_g^2 \to 0$ or as $\eta_w \to \eta_r$. We can thus approximate

$$\gamma - S_w \approx \frac{\gamma(1-\gamma)\sigma_g^2\left(\frac{1}{\eta_r} - \frac{1}{\eta_w}\right)}{\left(\frac{\gamma}{\eta_w} + \frac{1-\gamma}{\eta_r}\right)^2}. \qquad \ldots(1.24')$$

(1.22) and (1.24) have one further important implication: although when $\sigma_g^2 = 0$, $\alpha$ is indeterminate (all contracts are " equivalent "), as $\sigma_g^2 \to 0$,

$$\alpha \to \frac{\eta_r/1-\gamma}{\frac{\eta_w}{\gamma} + \frac{\eta_r}{1-\gamma}} \equiv \alpha^* \qquad \ldots(1.25a)$$

and

$$\frac{\beta}{f/l} \to \gamma - \alpha^* = \frac{\gamma(1-\gamma)(\eta_w - \eta_r)}{\eta_w(1-\gamma)+\eta_r\gamma} \equiv \frac{\beta^*}{f/l}. \qquad \ldots(1.25b)$$

(1.25) also makes clear that we approach a pure wage system ($\alpha = 0$) if $\eta_r$ is small (relative to $\eta_w$) and a pure rental system ($\alpha = 1$) as $\eta_w$ is small (relative to $\eta_r$).

We wish to solve now for $\alpha$ and $\beta$ explicitly in terms of the parameters of the problem:

$$\alpha = \alpha^* - \frac{\gamma(1-\gamma)}{\eta_w\eta_r} \frac{\sigma_g^2\left(\frac{1}{\eta_r} - \frac{1}{\eta_w}\right)}{\left(\frac{\gamma}{\eta_w} + \frac{1-\gamma}{\eta_r}\right)^4}$$

$$\frac{\beta}{f/l} = \gamma - \alpha^* - \frac{\gamma(1-\gamma)\left(\frac{1}{\eta_r} - \frac{1}{\eta_w}\right)}{\left(\frac{\gamma}{\eta_w} + \frac{1-\gamma}{\eta_r}\right)^2}\left(1 - \frac{\eta_w\eta_r}{(\eta_w(1-\gamma)+\gamma\eta_r)^2}\right)\sigma_g^2. \qquad \ldots(1.26)$$

Thus, if $\eta_w = 2$, $\eta_r = \frac{1}{2}$, $\gamma = \frac{2}{3}$, and $\sigma_g^2 = \frac{1}{2}$, $\alpha^* = \frac{1}{3}$, $S_w \approx \frac{1}{2}$, $\alpha \approx \frac{1}{6}$.

*Comparative statics.* We now analyse the effect of changes in risk, in labour/land ratios, and of technical progress on the equilibrium of the system. In addition to the assumptions made in the previous analysis of small variance, we now assume constant relative risk aversion. We focus our remarks on the case where $\eta_w > \eta_r$ (so $\beta > 0$); the other case follows symmetrically.

(*a*) *Changes in Risk* ($\sigma_g^2$)

From (1.24)-(1.26) it is clear that

**Proposition 5a.** *An increase in risk lowers* (*raises*) $\alpha$, *lowers* (*raises*) $S_w$ *and may raise or lower* $\beta$ *if* $\beta > (<)\ 0$. $\beta$ *is lowered* (*raised*) *when* $\gamma \leqq (\geqq) \frac{1}{2}$ *and* $\beta > (<)\ 0$.

(*b*) *Effects of Changes in the Labour/Land Ratio*

For simplicity, we focus on the effects on ($\alpha^*$, $\beta^*$), the limiting value of the distribution parameters. Straightforward calculations establish

$$\frac{d\alpha^*}{dl} = \frac{\eta_w\eta_r\gamma'}{(\eta_w(1-\gamma)+\eta_r\gamma)^2} \gtreqless 0 \text{ as } m \gtreqless 1$$

where $m$ = elasticity of substitution = $-f'(f-lf')/flf''$.

The percentage of worker's income received as fixed payments $\beta/\bar{Y}_w$, is given by

$$\frac{\bar{Y}_w}{\beta} \approx 1+\frac{\eta_r}{(1-\gamma)(\eta_w-\eta_r)}=\frac{(1-\gamma)\eta_w+\gamma\eta_r}{(1-\gamma)(\eta_w-\eta_r)}$$

so

$$\frac{d(\bar{Y}_w/\beta)}{dl}=-\frac{\eta_r\gamma'}{(\eta_w-\eta_r)(1-\gamma)^2} \gtrless 0 \text{ as } \beta(m-1) \lesseqgtr 0.$$

The actual change in $\beta$ is more complicated, because when $l$ increases, $\bar{Y}_w$ falls, so even if $\beta/\bar{Y}_w$ increases, $\beta$ may actually fall: ($\sim$ denotes "is of the same sign as")

$$\frac{d\beta}{dl} \sim \gamma^2\eta_r-(1-\gamma)^2\eta_w-m\gamma\eta_r$$

$$\sim \frac{-(1-\gamma)}{m}-(1-1/m)\frac{\gamma\eta_r}{\gamma\eta_r+(1-\gamma)\eta_w}.$$

Note that $d\beta/dl<0$ if $m \geqq 1$; but that if $m$ is sufficiently small, even though total mean wage payments are decreasing, the fixed part is increasing. Alternatively, if $\gamma \leqq \frac{1}{2}$ and $\eta_w>\eta_r$, then regardless of the value of $m$, $d\beta/dl<0$. We thus have

**Proposition 5b.** *If the variance is small, an increase in the labour-land ratio increases (decreases), $\alpha$ increases (decreases) the proportion of workers' income received in the form of fixed payments, and increases (decreases) the share of (mean) national income received by workers if the elasticity of substitution is greater (less) than unity.*

That is, as the workers become relatively better (worse) off, even though the degree of relative risk aversion is constant, the workers absorb proportionately more (less) of the risk although at the same time the proportion of their income received in fixed payments also increases (decreases).

(*c*) *Technical Change*

Consider the more general production function

$$Q=F(\lambda(\tau)L,\ \mu(\tau)T),$$

where $\lambda$ is the rate of labour augmentation and $\mu$ is the rate of land augmentation. Then

$$\frac{d\alpha^*}{d\tau} \gtrless 0 \text{ as } (m-1)\left(\frac{\lambda'}{\lambda}-\frac{\mu'}{\mu}\right) \gtrless 0$$

$$\frac{1}{\beta^*}\frac{d\beta^*}{\lambda}=\frac{\lambda'}{\lambda} \text{ for a Hicks neutral change (when } \lambda'/\lambda=\mu'/\mu)$$

$$=\frac{\lambda'}{\lambda}+\frac{1}{\beta^*}\frac{d\beta^*}{dl} \text{ for a Harrod neutral change}$$

$$\sim -\frac{d\beta^*}{dl} \text{ for a pure land augmenting invention.}$$

**Proposition 5c.** *$\alpha^*$ increases or decreases as technical change is land or labour saving in the Hicksian sense. $\beta$ increases for a Hicks neutral change, but may increase or decrease in other cases.*

### 1.3. *Diverse Individuals: Both Landlords and Workers Mix Contracts*

Essentially all the results of the preceding sub-section can be extended in a straightforward manner to the case where there are diverse individuals if both workers and landlords can mix contracts.

We argued in Section (1.1) that if both landlords and workers can " mix " contracts, the set of contracts $(x, \beta)$ must lie along the straight line defined by (1.13). Although not all points on the line are available in the form of a single contract, by mixing contracts, the individual can get any point [1] along the line.

Thus the $j$th landlord (who owns $T_j$ units of land)

$$\underset{\{x,\, l\}}{\text{maximizes}}\; EU_r^j[((f-xl)g-bl+axl)T^j]$$

so

$$EU_r^{j\prime}(-lg+al)T^j = 0 \qquad \ldots(1.27)$$

$$EU_r^{j\prime}[((f'-x)g-\beta)T^j] = 0$$

or

$$\frac{EU_r^{j\prime}g}{EU_r^{j\prime}} = a = \frac{\beta}{f'-x} \text{ for all } j \qquad \ldots(1.28)$$

or

$$f' = b/a \text{ for all } j. \qquad \ldots(1.29)$$

We have thus established

**Proposition 6a.** *The economy is productively efficient, i.e. all landlords use the same land/labour ratio.*

From (1.8), (1.13), and (1.28) we obtain

$$\frac{EU_w^{j\prime}g}{EU_w^{j\prime}} = a = \frac{EU_r^{j\prime}g}{EU_r^{j\prime}} \qquad \ldots(1.30a)$$

$$f' = \bar{Y}_w - \beta\left(1 - \frac{EU_w^{j\prime}}{EU_w^{j\prime}g}\right). \qquad \ldots(1.30b)$$

Equations (1.30) are identical to equations (1.20) and it is thus apparent that Propositions 2 and 3 apply here without modification.

The determination of the equilibrium values of $b$ and $a$ can be shown diagrammatically in Figure (1.5), using the Edgeworth-Bowley box introduced earlier. Since in equilibrium, the labour/land ratio must be the same on every farm, it must be the same as the aggregate labour/land ratio. This determines the mean and standard deviation of income. We assume there are two groups of equal size of labourers and one kind of landlord. Thus the point chosen by the landlord ($E$) must lie halfway between the points chosen by the two groups of workers ($E'$ and $E''$).

A higher value of $a$ would [2] decrease the landlord's demand for " risky " (from the workers' viewpoint) contracts, but decrease the supply of risky contracts by both groups of workers as illustrated in Figure (1.5). Hence, it is not an equilibrium.

If there are only a limited number of contracts (say because there are only two kinds of workers), differences in attitudes towards risk by landlords are reflected not in the contract they sign with any particular individual but in the mix of individuals they have on the farm. This is illustrated in Figure (1.6). The large landlord can " mix " workers with different risk attitudes. Although under our assumptions there is no risk diversification from mixing, since the returns to each plot of land (each farmer) are perfectly correlated, this " mixing " allows, in effect, workers with different risk attitudes to " trade " risks with one another through the intermediation of the landlord.

Indeed, it is easy to establish

[1] If $\beta_{max}$ is the " maximum " $\beta$ available in a single contract and $\beta_{min}$ the " minimum " $\beta$, then

$$\beta_{min} \leqq \beta \leqq \beta_{max}, \text{ and } (q-\beta_{max})/p < x < (q-\beta_{min})/p.$$

[2] From (1.29), given $l$ we know $b/a$.

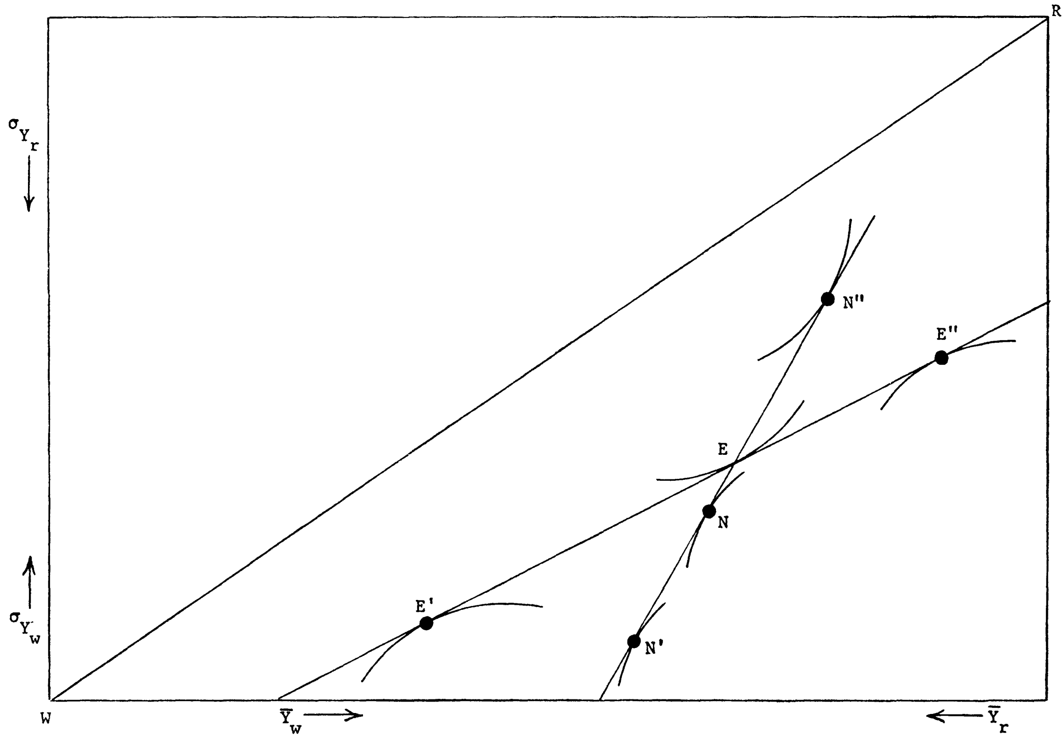


FIGURE 1.5

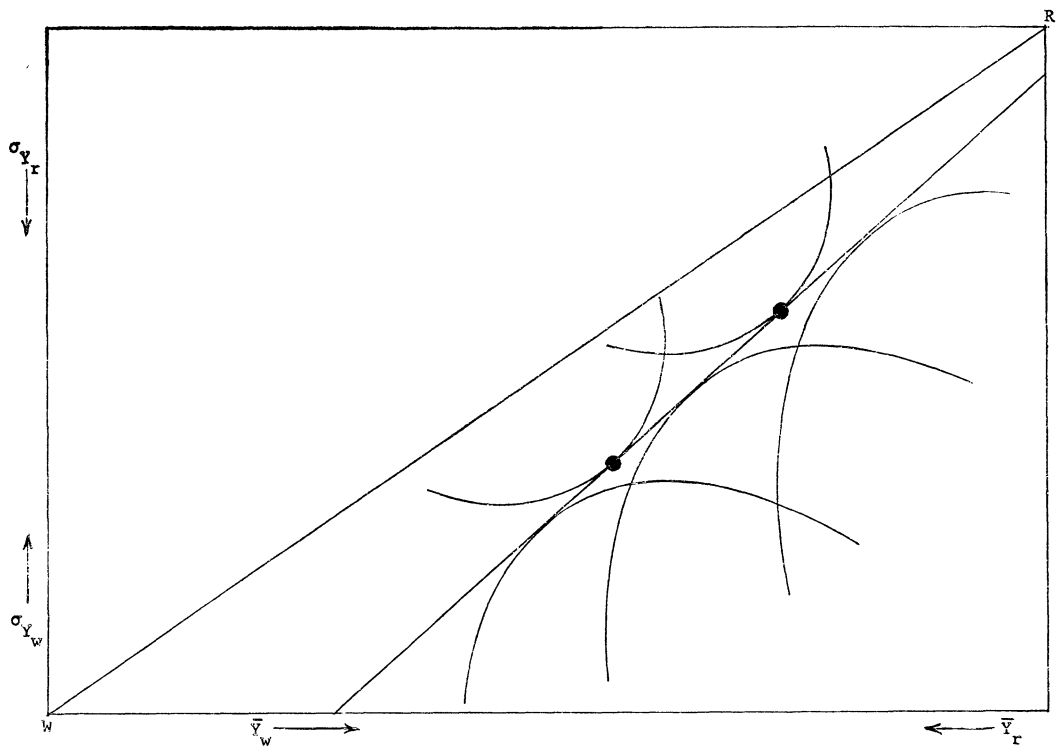


FIGURE 1.6

**Proposition 6b.** *If both landlords and workers can mix contracts, only two contracts are ever required. One of these is either the pure wage contract or the pure rental contract. If there is a pure sharecropping contract, it could have been dispensed with, i.e. all the relevant opportunities can be generated by mixing pure wage and pure rent contracts.*

These results follow upon observing that the slope of the diagonal is $\sigma g$ while the slope of the " contract line " $= \dfrac{\sigma\hat{g}}{1-a} > \sigma\hat{g}$ (using (1.15), (1.30) and (1.17a). Thus the contract line must intersect either the bottom or the top of the Edgeworth-Bowley box, and if it crosses the diagonal, it must intersect both.

1.4. *Diverse Individuals: Workers cannot mix Contracts*

The more interesting—and perhaps realistic—case is where workers cannot mix contracts. Then, as we argued in Section 1, the set of contracts available to the landlord could be described by

$$\beta = \beta(x) \text{ with } \beta' < 0,\ \beta'' \geqq 0.$$

The first order conditions for landlords' maximization can then be written as

$$\frac{EU_r^{j'}g}{EU_r^{j'}} = -\beta' = \frac{\beta}{f'-x}.$$

Thus

$$\frac{dl}{dx} = \frac{\beta\beta''}{\beta'^2 f''} \lesseqgtr 0 \text{ as } \beta \gtrless 0$$

$$\frac{dl}{d\alpha} = \frac{\dfrac{f}{l}\dfrac{dl}{dx}}{\dfrac{dl}{dx}\dfrac{x}{l}(1-\gamma)+1} \geqq 0 \text{ if } \beta \leqq 0,$$

where $\gamma = f'l/f =$ share of labour in the absence of uncertainty. If $\beta > 0$, $dl/d\alpha$ may be of either sign. Since less risk averse landlords choose contracts with a high $\beta$, and a lower $x$, we have established:

**Proposition 7.** *The presence of even a perfect rental market for land—in the absence of a perfect risk market—will not necessarily lead to productive efficiency for the economy. The more risk averse landlords who are less risk averse than their workers* (*so* $\beta > 0$) *have fewer workers per acre. The more risk averse landlords who are more risk averse than their workers* (*so* $\beta < 0$) *have more workers per acre, and their workers receive a larger share* $\alpha$. *Note, however, that all landlords signing a particular contract* $\{\beta, x\}$, *employ the same land/labour ratio.*[1]

Proposition 7 may provide part of the explanation of the phenomenon, noted in a number of LDC's, that there are sizeable variations in output per acre on different farms,[2] and if, as we might expect, risk aversion is systematically related to the wealth of the landlord (the size of his farm), we would expect output per acre to vary systematically with the size of farm.

[1] As long as there are only a finite number of types of individuals, $\beta(x)$ will be a piecewise linear function. The necessary modifications to the analysis are obvious.

[2] However, we should also expect to see these variations in output per acre to be systematically related to the kind of contracts signed. In practice, the variations of contracts in use at any point of time in a given economy probably have been much more limited than the variations in the contracts that have been employed over time. This may be either because, at any point of time, divergences in attitudes towards risk among landlords or among workers may be relatively small, or because preferences are of far less importance than the basic underlying technological considerations, or because the extreme assumptions of mobility which we employ were not satisfied in traditional societies. None the less, there is usually *some* range of choice (i.e. some sharecropping arrangement, a rental arrangement, or a wage arrangement).

There is, in these circumstances, an incentive for the creation of a " stock market " for farms. If there were a " stock market ", in which different landlords could buy shares in each others farms, in equilibrium the value of an acre would have to be the same regardless of the contract signed; hence the price of a " share " is the same regardless of the farm. We wish to know, will the introduction of this stock market result in production efficiency?

We are primarily interested in this question not for its implications for agricultural policy—with the exception of a few large American farms, shares in farms are not generally traded, probably because of the high " costs of information " which we have not taken into account here—but for its possible relevance to the " shares market " in modern capitalist economies. There has been some controversy over whether a stock market (without the full set of Arrow-Debreu securities) will in general lead to the correct investment decisions being made. In the particular model presented here, we shall now establish that

**Proposition 8.** *The economy will be productively efficient with a stock market—even though workers cannot sell shares in their own " wage " income.*

To see this, we must rewrite the income equations for landlords. Let $\omega^j$ be the $j$th farmer's ownership share on the $i$th farm. For simplicity, we assume all farms are of the same size. Then

$$Y_r^j = \Sigma\omega_i^j(f(l_i)(1-\alpha_i)g-\beta_i l_i), \quad \Sigma\omega_i^j = 1$$

where $\{l_i, \alpha_i, \beta_i\}$ is the " contract " signed by the $i$th farm with its labourers.[1] Hence, expected utility maximization results in

$$\frac{EU_r'^j g}{EU_r'^j} = \frac{\beta_i l_i}{(1-\alpha_i)f(l_i)} \text{ all } j \text{ and } i \text{ (for which } 0<\omega_i^j<1). \qquad \ldots(1.31)$$

If $\omega_i^j = 0$,

$$EU_r'^j[(1-\alpha_i)f(l_i)g-\beta_i l_i] \leqq 0 \qquad \ldots(1.31a)$$

and if $\omega_i^j = 1$

$$EU_r'^j[(1-\alpha_i)f(l_i)g-\beta_i l_i] \geqq 0. \qquad \ldots(1.31b)$$

Hence, it is clear for an " interior solution "

$$\beta = -\left(x-\frac{f(l)}{l}\right)\left(\frac{EU_r'^j g}{EU_r'^j}\right).$$

From (1.31) we can establish that $EU'^j g/EU'^j$ must be the same regardless of $j$. If two individuals hold shares in the same farm, this would follow immediately from (1.31). On the other hand, assume, for instance, that no individual holds shares in both farm 1 and 2. Label an individual owning shares in 1 with superscript 1, and similarly for farm 2. Then

$$\frac{\beta_2 l_2}{(1-\alpha_2)f(l_2)} \geq \frac{EU_r'^1 g}{EU'^1} \geq \frac{\beta_1 l_1}{(1-\alpha_1)f(l_1)}$$

and

$$\frac{\beta_1 l_1}{(1-\alpha_1)f(l_1)} \geq \frac{EU_r'^2 g}{EU_r'^2} \geq \frac{\beta_2 l_2}{(1-\alpha_2)f(l_2)}$$

from which it immediately follows that

$$\frac{EU_r'^j g}{EU_r'^j} = \frac{\beta_i l_i}{(1-\alpha_i)f(l_i)} \text{ all } i \text{ and } j.$$

[1] For simplicity, we have assumed each farm signs only one kind of contract. In the more general case, we would have

$$Y_r^j = \Sigma\omega_i^j(\Sigma z_i^k((f(l_i^k)-x_i^k)g-\beta_i^k l_i^k))$$

with $\Sigma z_i^k = 1$.

It is easy to show then that the only contracts signed will entail

$$-\beta' = \frac{EU_w'^k g}{EU_w'^k} = \frac{EU_r'^j g}{EU_r'^j} \text{ all } j, k$$

i.e. $\beta$ is a linear function of $x$—and hence as before the economy will be efficient.[1]

Although we have argued that in the absence of a stock market, the economy need not be productively efficient, it can be shown that there exists a competitive equilibrium which is productively efficient. Consider the stock market equilibrium. Recall that

$$Y_r^j = \Sigma\omega_i^j[(f_i - l_i x_i)g - \beta_i l].$$

But the same equilibrium could have been obtained if the farmer took his own plot of land and divided it up in proportions given by $\{\omega_i^j\}$ for each of which he hired labourers with the appropriate contract.[2] (This follows from our assumption of constant returns to scale; if there were any indivisibilities, this would not be possible.)

I had originally thought that when the economy was not productively efficient, there would be incentives for " mergers ", for a new farm to " integrate " the farms which previously had been owned by farmers with different risk attitudes. This does not in fact turn out to be the case—unless the merged farm can issue financial instruments other than safe bonds and common shares.

### 1.5. *Choice of Techniques and Crops*

Until now, we have assumed that the only decisions that the landlord has to make are (*a*) the land/labour ratio he uses and (*b*) the kind of contracts he signs. There are, of course, a number of other decisions to be made: what crop to grow, irrigation techniques, fertilization techniques, etc. A number of considerations are involved in making these decisions: the pattern of allocation of labour over the year, required levels of fixed investment, and, in an economy in which there is only limited trading of commodities, the diversity of food in the diet. As Boserup has argued some crops and techniques seem more suited for low density situations, others for high densities. But for our purposes, we are concerned primarily with the implications that these choices have for the pattern of returns across the states of nature.

We no longer will assume, in other words, that $g(\theta)$ is given. Indeed, we will not even assume that there is "multiplicative uncertainty". The interest in this question arises in part from the fact that in other models without a full set of risk markets which analyse the problem of choice of technique, where there is not multiplicative uncertainty, the choice is not Pareto optimal. (See Stiglitz [14] and Jensen and Long [11].) Here we shall be able to establish that

**Proposition 9.** *If there exists a linear relationship between the workers' mean income from a contract and the standard deviation of the income from the contract, then the choice of techniques will be Pareto optimal.*

Both for purposes of simplicity and to make our analysis directly comparable to earlier models referred to above, we shall employ in this section mean variance analysis. We shall assume that both landlords and workers evaluate alternative income patterns in terms of their means and standard deviations

$$EU_r = u_r(\mu_r, \sigma_r) \qquad \ldots(1.32a)$$

$$EU_w = u_w(\mu_w, \sigma_w) \qquad \ldots(1.32b)$$

where $\mu_r(\mu_w)$ is the mean income of the landlord (worker) and $\sigma_r(\sigma_w)$ is its standard deviation.

[1] A similar analysis would have obtained if we allowed workers to trade shares in their fixed income.
[2] Assuming all farms are sufficiently large so that the indivisibility of labour is no problem.

We shall assume that the mean and standard deviation of the output of a worker is a function of the land/labour ratio, $l$, and the choice of technique, which we shall denote simply by $k$.

$$\mu = \mu(l, k)$$

$$\sigma = \sigma(l, k).$$

For simplicity assuming there is a single landlord, but several kinds of labour,[1] which we denote by subscript $i$;

$$\frac{\mu_r}{T} = \Sigma\mu(l_i, k_i)\phi_i l_i(1-\alpha_i) - \beta_i l_i \phi_i$$

$$= \Sigma\mu(l_i, k_i)\phi_i l_i - \mu_{w_i} l_i \qquad \ldots(1.33)$$

$$\frac{\sigma_r^2}{T^2} = \sum_i \sum_j (1-\alpha_i)\sigma_{ij}(l_i, l_j, k_i, k_j)(1-\alpha_j)\phi_i\phi_j l_i l_j \qquad \ldots(1.34)$$

where $\phi_i$ is the percentage of land allocated to the $i$th-type labour $\sigma_{ij}$ is the covariance between output (per man) of $i$th and $j$th kinds of workers, assumed to be a function only of $l_i$, $l_j$, $k_i$, and $k_j$.
Similarly,

$$\mu_{w_i} = \alpha_i\mu(l_i, k_i) + \beta_i$$

$$\sigma_{w_i} = \alpha_i\sigma_{ii}(l_i, k_i).$$

The problem of the competitive farm is to

$$\max_{\{l_i, k_i, \phi_i, \mu_{w_i}, \sigma_{w_i}\}} u_r(\mu_r, \sigma_r)$$

subject to (1.15).

The problem of Pareto optimality may be written

$$\max_{\{l_i, k_i, \phi_i, \mu_{w_i}, \sigma_{w_i}\}} u_r(\mu_r, \sigma_r)$$

subject to $u_{w_i}(\mu_{w_i}, \sigma_{w_i}) = \bar{u}_{w_i}$.

If we form the corresponding Lagrangeans,

$$\mathscr{U} = u_r(\mu_r, \sigma_r) + \lambda(p\sigma_{w_i} + b - \mu_{w_i})$$

and

$$\hat{\mathscr{U}} = u_r(\mu_r, \sigma_r) + \Sigma\hat{\lambda}_i(\bar{u}_{w_i} - u_w(\mu_{w_i}, \sigma_{w_i}))$$

we obtain the same expressions for the derivatives with respect to $k$, $l$, and $\phi$ (since neither involve the constraints). Thus any differences must arise in one of the following expressions:

$$\frac{\partial\mathscr{U}}{\partial\mu_{w_i}} = \frac{\partial u_r}{\partial\mu_{w_i}} - \lambda = 0 \qquad \frac{\partial\mathscr{U}}{\partial\sigma_{w_i}} = \frac{\partial u_r}{\partial\sigma_{w_i}} + p\lambda = 0 \qquad \ldots(1.34a)$$

$$\frac{\partial\hat{\mathscr{U}}}{\partial\mu_{w_i}} = \frac{\partial u_r}{\partial\mu_{w_i}} - \hat{\lambda}_i\frac{\partial u_{w_i}}{\partial\mu_{w_i}} = 0 \qquad \frac{\partial\hat{\mathscr{U}}}{\partial\sigma_{w_i}} = \frac{\partial u_r}{\partial\sigma_{w_i}} - \hat{\lambda}\frac{\partial u_{w_i}}{\partial\sigma_{w_i}} = 0 \qquad \ldots(1.34b)$$

so from (1.34$a$) and (1.34$b$) we obtain, respectively.

$$-\frac{\partial u_r/\partial\mu_{w_i}}{\partial u_r/\partial\sigma_{w_i}} = \frac{1}{p}$$

$$\frac{\partial u_r/\partial\mu_{w_i}}{\partial u_r/\partial\sigma_{w_i}} = \frac{\partial u_{w_i}/\partial\mu_{w_i}}{\partial u_{w_i}/\partial\sigma_{w_i}}.$$

[1] This assumption is adopted only for notational simplicity.

The two are equivalent if

$$\frac{1}{p} = \frac{\partial u_{w_i}/\partial \mu_{w_i}}{\partial u_{w_i}/\partial \sigma_{w_i}},$$

which will be true if workers choose the contract which maximizes utility.

The assumption about the linearity of the relationship between mean and standard deviations of contracts offered to workers is crucial. If this assumption is not satisfied, we already noted that the economy may not be productively efficient; different farms will have different labour/land ratios. Here we note an additional complication.

In the previous sections the factors " land " and " labour " played a perfectly symmetrical role in the analysis. This may no longer be true when, as returns to different plots of land and different workers are not perfectly correlated and workers and/or landlords cannot mix contracts. The question may be put as follows: assume each worker makes the decision of the choice of technique. He may " negotiate " with the landlord about which crops to grow, etc., but in the negotiations, the landlord takes as given his contracts with all other workers. Will the equilibrium which emerges be the same as that which would arise if the landlord made all the decisions? The answer is not necessarily. (For an example of the misallocation resulting from failure to " coordinate ", see Stiglitz [15].)

## PART II. ELASTIC LABOUR SUPPLY

### 1. *Introduction*

In the traditional literature, sharecropping arrangements have been criticized in terms of their incentive effects on the supply of labour. In the previous analysis, we focused on the risk sharing aspects of sharecropping, and explicitly assumed that the labour supply was inelastic. On the other hand, there is a fundamental objection to the usual analyses of sharecropping: they fail to take account of the original motivations in using a sharecropping arrangement rather than a wage or rental system. Thus, if the motivation is risk sharing, as we have described it in the previous sections, then in general, the average income received by a worker will not be equal to his marginal product; an analysis of the " efficiency " of the sharecropping system must explicitly take into account the attitudes towards risk of the workers and landlords, and how this affects their behaviour. Clearly, a full analysis of a sharecropping economy must take into account both incentive and risk-sharing effects, and that we propose to do in the next two sections. In Section 2 we assume that contracts specify precisely the amount of labour to be provided and that they are enforceable without cost, while in Section 3, we examine the other polar case where the contract makes no specification concerning the labour to be supplied by the worker.

### 2. *Enforceable Contracts*

The landlord is interested in the amount of labour that will be supplied to any piece of land; he is not particularly interested in the number of labourers on a piece of land. Thus, contracts between landlords and their labourers will specify the amount of labour that a worker is to provide. (But this in itself is not sufficient to guarantee that the correct optimality conditions will be satisfied.)

Thus, there are two decisions for the worker to make: what kind of contract to sign, and how much labour to supply. A contract, recall, is specified by an assignment of labour per acre, $l$, a fixed fee, $\beta$, and a share, $\alpha$. We assume labourers can " mix contracts ", Hence, as we argued above, in equilibrium, labour (not however *labourers*) per acre is the same everywhere, and so can be taken as given by the worker; and we can take the fixed fee to be a linear function of the share

$$\beta = -a\frac{\alpha f(l)}{l} + b. \qquad ...(2.1)$$

Thus a worker's maximization problem is to

$$\max_{\{\alpha,\, L\}} V(L)+EU_w[(\alpha g+\beta)L], \qquad \ldots(2.2)$$

where $L$ is the supply of labour by a labourer and where we have assumed for simplicity that workers have an additive utility function and $V'<0$, $V''<0$. Substituting (2.1) into (2.2) we obtain

$$\max V(L)+EU_w\{(x(g-a)+b)L\}. \qquad \ldots(2.2')$$

The first-order conditions are thus

$$V'+EU_w'\left\{\frac{\alpha f(l)}{l}g+\beta\right\}=0 \qquad \ldots(2.3a)$$

$$EU_w'(g-a)=0. \qquad \ldots(2.3b)$$

The second condition is just another form of the familiar condition we have already discussed above [equation (1.8)]. Here our concern is with (2.3$a$). $\alpha(f(l)/l)g+\beta$ is just the return per unit of labour. (2.3$a$) says that the marginal disutility of labour should be just equal to the expected marginal utility of the income received as a result.

To compare this with the optimal allocation, we consider a " command " economy in which the same risk sharing possibilities exist but in which the decision on the supply of labour is made centrally. Then, if all workers are identical and landlords are identical (the general case follows along identical lines, but is slightly more complicated notationally) we wish to

$$\max EU_r[(1-\alpha)f(L)g-\beta L] \qquad \ldots(2.4)$$

where we have normalized the supply of land and of labourers at unity, subject to the constraint that

$$V(L)+EU_w[\alpha f(L)g+\beta L]=\bar{U} \qquad \ldots(2.5)$$

where we have normalized the supply of land at unity. We then obtain as first order conditions

$$EU_r'g=vEU_w'g \qquad \ldots(2.6a)$$

$$EU_r'=vEU_w' \qquad \ldots(2.6b)$$

$$EU_r'[(1-\alpha)f'g-\beta]+vEU_w'(\alpha f'g+\beta)+vV'=0 \qquad \ldots(2.6c)$$

where $v$ is the Lagrange multiplier associated with the constraint (2.5).

Using (2.6$a$) and (2.6$b$), we can rewrite the condition (2.6$c$) as

$$V'+EU_w'gf'=0.$$

This should be compared with (2.3$a$), which may be rewritten, using (1.29) and (2.3$b$), also to read

$$V'+EU_w'gf'=0$$

i.e. the competitive supply of labour is optimal. The divergences between expected marginal products of labour and the marginal disutility of labour are just those which are " optimal " given the attitudes towards risk of the two groups. We have thus established

**Proposition 10.** *If labour is elastically supplied, and contracts are enforceable without cost, landlords will specify the amount of labour to be supplied by the sharecropper in the sharecropping contract and, if both landlords and workers can mix contracts, the equilibrium will be Pareto optimal.*

## 3. *Incentive Effects*

### 3.1. *The Basic Model*

It is curious that economists, in discussing the misallocation of resources resulting from a sharecropping arrangement, have focused on the *undersupply* of labour (" negative incentive ") in the sharecropping system, while businessmen talk about the positive incentive effects of " sharing " arrangements (e.g. commissions) as one of their major advantages. The reason for the discrepancy lies in the economists' conventional simplifications of the production process. There is a " book of blueprints " (a recipe book), with a simple set of instructions on how to combine two homogeneous factors to produce the maximum output. There are no unexpected circumstances, no contingencies in the " field " to be taken account of, and no difficulty in ascertaining whether a given individual has followed the instructions in the book of blueprints. The " contract " between the worker and the employer specifies that the worker will follow the appropriate page in the book for blueprints, and if he fails to do so, he receives no compensation. For some purposes, this is a useful simplification; for others—and in particular for the purpose of understanding the nature of the contract signed between landlord and worker—it is not.

To describe the input provided by a labourer, we must not only know the number of hours he works, but also what we shall loosely call, the " effort " of the individual. As difficult as it may be to ascertain the former (in an agricultural situation), it is even more difficult to ascertain the latter. " Effort " affects output in a number of ways: first, and most obvious, is the *pace* at which an individual works, say the number of weeds removed per hour. If this were the only consideration, we could simply measure labour supplied by the corresponding output (a piece-work system) and not by the hour. Secondly, there is the *thoroughness*; say the completeness with which a given acre is weeded. Two individuals may pick the same number of weeds per hour, but one picks the most obvious weeds in a two-acre plot, while the other picks all the weeds in a one-acre plot. The point of course is that there are different costs associated with picking different weeds, and that is one of the reasons that pay cannot be simply proportional to the number of weeds picked. But it is difficult except by very close supervision to ascertain the cost associated with each weed. Thirdly, there is the *efficiency* of the individual, which is equivalent to the individual equating the marginal cost of a weed picked on each plot of land.[1] This is even more difficult to ascertain than the thoroughness. Fourthly, there is the efficiency of decision making under uncertainty: if it rains right before harvest, will the farmer make the " correct " decision about revising plans about harvesting and carry those plans out. Presumably, if all the possible contingencies, e.g. all the possible sequences of rain, sun, and visitations by insects and disease were spelled out in the " book of blueprints " this would not be a factor. In fact, however, only a few of the possible contingencies can be simply described, and the appropriate courses of action to take specified. Finally, there is the *inventiveness* of the individual: as he picks weeds, he may develop more efficient and better ways of doing it. These " inventions " may be specific to the specific circumstances of the given farm, and therefore are not likely to be discussed except in very broad terms in any general treatment of weeding. This list of " qualities " associated with labour is not meant to be exhaustive. One of the very important considerations when a third factor, such as capital, is present, is the care which the worker takes of the capital supplied by the landlord.[2]

A contract, accordingly, may not only specify the hours of labour to be provided, but also something about the *effort* required of the individual, the degree of " control " over the day-to-day decisions that are to rest in his hands and the amount of supervision (direction) he will receive from managers. There is also an implicit or explicit penalty-

[1] Assuming the marginal benefit is constant, or equating the net marginal benefit of a weed picked on each piece of land, if the marginal benefit is not constant.

[2] In the absence of uncertainty, we could infer the level of input from the level of output; with uncertainty, this may not be possible. See [15].

rewards function. A piece-work wage and a commission for sales beyond a certain level are examples of rewards; being fired is an example of a penalty.

Contracts clearly vary in the preciseness with which the labour inputs (effort) required of the worker are specified, and in the extent and manner in which those terms are enforced. In this section we consider the polar case to that analysed in the previous one: there we implicitly assumed that the contract specified everything, here we assume that the contract specifies nothing concerning the amount (quality) of labour to be provided.[1] The reward to greater effort is thus provided by his share of the output ($\alpha$). Both models are polar cases, but given the complexity of completely specifying (and enforcing) the labour inputs, as we have described them above, it may be argued that the model of this section is the better approximation to reality.

We assume the landlord is risk neutral, but that the worker is not.[2] Without incentive effects, the landlord would absorb all the risk; workers would all face a pure wage contract. We can show now, however, that there will still be some sharecropping (i.e. $\alpha > 0$).

We return to our simpler model in which all individuals are identical. The worker's utility function is of the form,[3, 4]

$$U = EU[Y_w] + V(e), \qquad \ldots(3.1)$$

where $V' < 0$ is the marginal disutility from an increment in $e$, " effort " or " effective labour supply per individual ". If this worker faces a contract ($\alpha$, $\beta$, $l$) he chooses $e$ to maximize $U$. Now $l$ refers to the density of *workers*. An increase in $e$ increases $Q$, so if $\alpha > 0$, it increases $Y_w$. Assume that effort is " purely labour augmenting " [5] Then we have

$$Q = Tf(el) \qquad \ldots(3.2)$$

and

$$Y_w = \frac{\alpha f(el)}{l} g + \beta \qquad \ldots(3.3)$$

so, given $\alpha$, $l$, and $\beta$, $e$ is chosen so that

$$EU'_w \alpha f'(el) g + V' = 0. \qquad \ldots(3.4)$$

A worker is indifferent among contracts that yield the same level of $U$ when he has optimally chosen $e$. Thus, the set of utility equivalent contracts is given by

$$\overline{W} = \max_{\{e\}} EU\left[\frac{\alpha f(el)g}{l} + \beta\right] + V(e), \qquad \ldots(3.5)$$

which, as before, we can solve for $\beta$ as a function of $\alpha$ and $l$:

$$\beta = h(\alpha, l, \overline{W}) \qquad \ldots(3.6)$$

$$\frac{\partial \beta}{\partial \alpha} = -\frac{f}{l}\frac{EU'g}{EU'} < 0, \quad \frac{\partial \beta}{\partial l} = \frac{\alpha(f - f'el)}{l^2}\frac{EU'g}{EU'} > 0. \qquad \ldots(3.6a)$$

[1] Presumably because, even if there were specifications, the contract would not be enforceable because of prohibitive supervisory costs.

[2] Thus, in this section we can drop the subscript $w$ on $U_w$ without ambiguity.

[3] It should be clear that our analysis of incentive effects and " effort " is closely related to Leibenstein's important work on $X$-efficiency.

[4] " Effort " appears in this formulation exactly like " labour " in Section 2.

[5] Akerlof has argued that improvements in labour effort by reducing, for instance, damage to machines, is capital augmenting [1].

Different contracts, which are equivalent in utility terms to the worker, have different incentive effects: from (3.4) and (3.6) we obtain

$$\left(\frac{\partial \ln e}{\partial \ln l}\right)_W = -\frac{E\left\{U''\alpha f'g\left[\frac{\alpha(f-f'el)}{l}\left(\frac{EU'g}{EU'}-g\right)\right]+U'\alpha f''elg\right\}}{E\left\{U''\alpha f'\frac{(\alpha f'el)}{l}g^2+U'\alpha f''elg\right\}+V''} \qquad ...(3.7a)$$

$$\left(\frac{\partial \ln e}{\partial \ln \alpha}\right)_W = -\frac{E\left\{-U''\alpha f'g\left[\frac{\alpha f}{l}\left(\frac{EU'g}{EU'}-g\right)\right]+U'\alpha f'g\right\}}{E\left\{U''\alpha f'\frac{(\alpha f'el)}{l}g^2+U'\alpha f''elg\right\}+V''e}. \qquad ...(3.7b)$$

The landlord wishes to maximize,

$$\max_{\{\alpha,\, l\}} (1-\alpha)f(el)-hl, \qquad ...(3.8)$$

which yields the first-order conditions

$$-f-l\frac{\partial h}{\partial \alpha}+(1-\alpha)f'l\left(\frac{\partial e}{\partial \alpha}\right)_W=0 \qquad ...(3.9a)$$

$$\left(e+l\left(\frac{\partial e}{\partial l}\right)_W\right)(1-\alpha)f'-h-l\frac{\partial h}{\partial l}=0. \qquad ...(3.9b)$$

(3.9) may be rewritten as

$$\alpha=\frac{\gamma\left(\frac{\partial \ln e}{\partial \ln \alpha}\right)_W}{c+\gamma\left(\frac{\partial \ln e}{\partial \ln \alpha}\right)_W},\quad \frac{S_w-\gamma}{\gamma}=\frac{c\left(\left(\frac{\partial \ln e}{\partial \ln l}\right)_W+(1-\gamma)\left(\frac{\partial \ln e}{\partial \ln \alpha}\right)_W\right)}{c+\gamma\left(\frac{\partial \ln e}{\partial \ln \alpha}\right)_W}, \qquad ...(3.10)$$

where $c \equiv 1-EU'g/EU' \geqq 0$, with equality only if $U''=0$ or $\alpha=0$. From (3.10) we can establish

**Proposition 11.** *If workers are risk averse, then* $0<\alpha<1$, *and* $\alpha$ *is larger the greater is the responsiveness of effort to an increase in the share.* $\alpha$ *is smaller the more risk averse the individual. For a given response elasticity and degree of risk aversion,* $\alpha$ *is greater the greater is* $\gamma$ *(the share of labour in the absence of uncertainty). If workers are risk neutral,* $\alpha=1$.

**Proposition 12.** *If workers are risk averse, they receive a mean income greater than, equal to or less than their mean marginal product as the elasticity of substitution is greater than, equal to, or less than unity. If workers are risk neutral, they receive on average their mean marginal product.*

This proof of Proposition 11 is straightforward;[1] proposition 12 follows from observing that

$$S_w-\gamma \gtreqless 0 \text{ as } -\frac{\left(\frac{\partial \ln e}{\partial \ln l}\right)_W}{\left(\frac{\partial \ln e}{\partial \ln \alpha}\right)_W} \lesseqgtr (1-\gamma)$$

[1] At $\alpha=0$, $c=\left(\frac{\partial \ln e}{\partial \ln \alpha}\right)_W=\left(\frac{\partial \ln e}{\partial \ln l}\right)_W=\frac{\partial h}{\partial l}=e=0$. (3.9*b*) is satisfied only if $\beta=0$.

and substituting (3.7) into the above.[1] As usual, $\beta \gtreqless 0$ as $S_w \gtreqless \alpha$. We have not been able to find simple conditions for determining whether $S_w \gtreqless \alpha$ except when the elasticity of substitution is unity. Then, from (3.10) $S_w = \gamma$; then

$$\beta \gtreqless 0 \text{ as } \left(\frac{\partial \ln e}{\partial \ln \alpha}\right)_{\overline{W}} \lesseqgtr \frac{c}{1-\gamma}. \quad \text{...(3.11)}$$

Earlier, we noted that if workers could not " mix " contracts, differences in attitudes towards risk would result in variations in output per acre. The economy would not be " productively efficient ". Similar variations in output per acre arise because of differences in the efficacy of incentives among different individuals.

We first observe that the landlord must get the same rent from each piece of land, i.e.

$$(1-\alpha)f-\beta l = \bar{R} \quad \text{(a constant)} \quad \text{...(3.12)}$$

Clearly,

$$f(el) = \frac{\bar{R}}{1-S_w}. \quad \text{...(3.13)}$$

Since $S_w$ will differ with different labourers, because $(\partial \ln e/\partial \ln l)_{\overline{W}}$, $(\partial \ln e/\partial \ln \alpha)_{\overline{W}}$, or $c$ differ, unless $m = 1$ (so $S_w = \gamma$), $f(el)$ will differ.

We have thus shown

**Proposition 13.** *Output per acre will in general differ on different farms: the economy will not be productively efficient unless the production function has unitary elasticity of substitution.*

Other things being equal, an increase in uncertainty increases $c$ and hence increases differences in $S_w$ arising from differences in the strength of incentive effects. Other things being equal, the more risk averse, the greater $c$ and hence the higher $S_w$ and output per acre.[2]

A question of some interest in LDC's is whether output per acre is higher or lower on more densely populated land. Conventional analysis ignores the effect of effort, so argues the greater $l$, the greater $el$, the greater $f(el)$; $e$ may, however, decrease with an increase in $l$, and if it decreases enough, $el$ is reduced.

### 3.2. *Comparison with Wage System*

We argued earlier that the conventional wisdom, that the sharecropping system results in too little supply of labour, errs in that it ignores the very reasons for introducing sharecropping. In Section 2 we showed that, when risk was taken into account, the supply of labour under sharecropping was in fact optimal with fully enforceable contracts. Here, we wish to make two further observations:

(1) It is not really meaningful—from a welfare point of view—to compare a situation where a particular kind of contract is enforceable at zero cost, with one in which it is not enforceable (or enforceable only at an infinite cost). By ignoring the costs of enforcing the contract, one is ignoring part of the essence of the problem.

(2) But even if we simply limit ourselves to comparing the allocation of resources, the conventional presumption does not appear to be correct. For it is a partial equilibrium analysis, and in going from one system to the other, the distribution of income as well as the sharing of risk will change. Since the switch to a wage system from a sharecropping system will in general change the individual's level of utility, the net result is a mixture of substitution and income effects.

[1] $\left(\frac{\partial \ln e}{\partial \ln \alpha}\right)_{\overline{W}}(1-\gamma)+\left(\frac{\partial \ln e}{\partial \ln l}\right)_{\overline{W}} \sim \alpha\left(\frac{f'(f-f'el)}{f}+f''el\right) EU'g.$

[2] For $m$ near unity. Otherwise, there may be large offsetting changes in $\gamma$.

We can compare this equilibrium with one where the contracts are enforceable (Section 2). Then $\alpha = 0$ (except in the case where there is no uncertainty when $\alpha = 1$ or $\alpha = 0$). We define $\beta^*$ as the price of a unit of effort. The worker chooses $e$ so that

$$V' + U'\beta^* = 0, \qquad \text{...(3.4')}$$

while the landlord sets (if he is risk neutral)

$$lf' = \beta^*. \qquad \text{...(3.9')}$$

Let superscripts $s$ and $w$ denote the variables for the sharecropping and wage systems respectively). There are two questions: (*a*) at $e = e^s$, is the increment in expected utility from additional effort greater or less under sharecropping than under the wage system, i.e. is

$$\alpha^s EU'\left[\frac{\alpha^s f(e^s l)g}{l} + \beta^s\right] g \gtrless U'[f'(e^s l)el^s]? \qquad \text{...(3.14)}$$

In the absence of uncertainty, the context in which these questions are usually investigated, the LHS of (3.14) always exceeds the RHS. That either inequality may hold may be seen by considering the Cobb-Douglas production function. By Proposition 12, $EY_w = f'e$, and hence by Jensen's inequality,

$$EU'Y_w \lesseqgtr f'elU'(f'el) \text{ as } \eta \lesseqgtr 1$$

and

$$f'eEU'g \gtrless EU'Y_w \text{ as } \beta \lesseqgtr 0.$$

It is clear that if $\eta < 1$, $\beta > 0$, the RHS of (3.14) exceeds the LHS. In other cases, the result is ambiguous.[1] (*b*) What is the effect of a change in $e$ on

$$U'\beta^* + V'?$$

The conventional partial equilibrium analysis observes that by the second order condition

$$U''\beta^{*2} + V'' < 0,$$

but general equilibrium analysis takes into account the change in $\beta^*$; the general equilibrium effect depends on the sign of

$$V'' + U''f'^2l^2 + U''f''el^3f' + U'f''l^2 \sim -(1-\eta)\left(\frac{1}{d\ln e/d\ln\beta} + \frac{1-\gamma}{m}\right), \qquad \text{...(3.15)}$$

where $m$ is the elasticity of substitution and $\eta = -U''\beta e/U'$.

We can thus establish that

**Proposition 14.**

*$e^w \gtrless e^s$ as*

$$\left\{\alpha^s EU'\left(\frac{a^s f(e^s l)}{l} + \beta^s\right)g - U'(f'(e^s l)e^s l)\right\}\left\{(\eta-1)\left(\frac{1}{d\ln e/d\ln\beta} + \frac{1-\gamma}{m}\right)\right\} \lesseqgtr 0.$$

There appears to be no presumption that sharecropping reduces effort (labour) from what it would be under a wage system with enforceable contracts.

[1] E.g. if $\eta = 2$, $\left(\frac{\partial \ln e}{\partial \ln \alpha}\right)_W = \frac{c}{1-\gamma}$, so $\beta = 0$, $\alpha = \gamma$. Let

$$g = \begin{cases} \frac{1}{2} \text{ with probability } 0{\cdot}5 \\ \frac{3}{2} \text{ with probability } 0{\cdot}5 \end{cases}$$

Then the LHS exceeds the right as $\gamma \gtrless \frac{3}{4}$.

3.3. *Choices of Techniques*

Unlike the earlier economies analysed, where even though workers and landlords had different attitudes towards risk, if both landlords and workers could mix contracts the sharecropping system was able to equate their marginal rates of substitution (say between mean and standard deviation of income), here this is not true. In our example in the previous subsections, landlords were risk neutral, while workers still bore a large part of the risk. Thus, in questions of choice of technique and crops, there will appear to be conflict of interest between the landlord and workers. At any specified share contract, the landlord wants only to have the worker choose whatever technique or crop maximizes expected output; the worker is willing at the margin to sacrifice some mean output for a reduction in risk. Although some of the " conflict " can be avoided by careful stipulation within the contract (although both sides may feel " unhappy " in the sense that they could imagine a better " contract " presumably neither side would sign if there existed other individuals with whom they could sign a better contract) for many of the same reasons that it is difficult to enforce a contract for a stated level of " effort ", requirements on technique of production may be difficult to enforce without close (and costly) supervision.

One of the most important choices in technique involves the use of other factors of production, in particular, of capital.

If, as in Sections 1-2, there were no problems in contract enforcement, a contract would be specified by the amount of capital contributed by both the landlord and the worker. It can be shown

**Proposition 15.** *With completely enforceable contracts, if contracts can be mixed, and if there is a safe investment yielding a return of $r$, the mean marginal product of capital is just $r/(EU'g/EU')$, and will be the same for all farms. If workers' contracts cannot be mixed, then the more risk averse individuals will have a higher mean marginal product of capital.*[1]

The more interesting and difficult problems arise when there are " enforcement " problems. First, the operations that need to be specified in the contract are likely to be increased, and hence the need for supervision is increased. Did the machine break down because the worker did not treat it correctly? Secondly, in the absence of close supervision, it is important to provide an incentive for the worker to use the capital goods supplied by the landlord appropriately. Thirdly, it is advantageous to the landlord (as long as he

[1] Let $k_r$ be the capital per acre contributed by the landlord, $k_w$ be the capital per acre contributed by worker. Then $\beta = \beta(\alpha, l, k_w, k_r)$. The worker maximizes

$$EU_w\left[\frac{\alpha f(l, k_w+k_r)}{l}g+\beta+r\left(\frac{k}{l}-\frac{k_w}{l}\right)\right]$$

so

$$\alpha f_k\frac{EU_w'g}{EU_w'} \geqq r-\beta_{k_w}l \qquad \text{...(3.16}a\text{)}$$

with equality always holding if workers can borrow;

$$\alpha f_k\frac{EU_w'g}{EU_w'} = -\beta_{k_r}l. \qquad \text{...(3.16}b\text{)}$$

Similarly, for the landlord, we obtain

$$(1-\alpha)f_k\frac{EU_r'g}{EU_r'} = \beta_{k_w}l \qquad \text{...(3.16}c\text{)}$$

$$(1-\alpha)f_k\frac{EU_r'g}{EU_r'} \geqq r+\beta_{k_r}l \qquad \text{...(3.16}d\text{)}$$

with equality always holding if landlords can borrow. Assume (3.16*a*) or (3.16*d*) hold with equality. If

$$\frac{EU_w'g}{EU_w'} = \frac{EU_r'g}{EU_r'}$$

then

$$f_k\frac{EU'g}{EU'} = r.$$

Note also that if either (3.16*a*) or (3.16*d*) holds with equality, then they both must hold with equality.

receives any share of the output) to have the worker supply as much of his own capital as possible, and conversely (for any given share-fixed fee contract) it is advantageous to the worker to have the landlord supply as much of the capital as possible.

We can use a simple modification of the model presented earlier to illustrate these points. The landlord, in deciding how much capital he should invest, needs to take account of the fact that, in general, the more capital he invests, the less his workers will invest; but the more capital he supplies, the higher the return to effort, and hence the greater effort he is likely to obtain from his workers. Thus, if he is risk neutral, he

$$\text{maximizes } (1-\alpha)f(el, k_w-k_r)-\beta(k_r, \alpha, l)l+r(\bar{k}-k_r) \qquad \text{...(3.17)}$$

so that

$$f_k-r \leqq \alpha f_k-\left[(1-\alpha)f_{el}l\left(\frac{\partial e}{\partial k_r}\right)_W-\frac{\partial \beta}{\partial k_r}\right] \qquad \text{...(3.17}a\text{)}$$

$$e(1-\alpha)f_{el}\left(1+\left(\frac{\partial \ln e}{\partial \ln l}\right)_W\right)-\beta-\frac{l\partial\beta}{\partial l}+(1-\alpha)f_k\left(\frac{\partial k_w}{\partial l}\right)_W=0 \qquad \text{...(3.17}b\text{)}$$

$$-f-\frac{l\partial\beta}{\partial\alpha}+(1-\alpha)\left[f_{el}l\left(\frac{\partial e}{\partial\alpha}\right)_W+f_k\left(\frac{\partial k_w}{\partial\alpha}\right)_W\right]=0. \qquad \text{...(3.17}c\text{)}$$

(3.17$a$) can be rewritten

$$f_k \leqq \frac{r-(1-\alpha)f_{el}l\left(\frac{\partial e}{\partial k_r}\right)_W}{1-\alpha c} \qquad \text{...(3.18}a\text{)}$$

with equality holding if the landlord does any investing. Similarly, if the worker does any investing

$$f_k=\frac{r}{\alpha(1-c)}. \qquad \text{...(3.18}b\text{)}$$

Hence we obtain

**Proposition 16.** *In contracts without supervision the capital is supplied by the worker or by the capitalist, depending on whether*

$$r+\alpha(1-c)f_{el}l\left(\frac{\partial e}{\partial k_r}\right)_W \lessgtr 0.$$

The case where $k_w = 0$ follows essentially along the lines developed earlier. On the other hand when $k_r = 0$, from (3.17$c$) it is clear that $\alpha$ will be larger than in the cases analysed earlier: in addition to the incentive effect on effort, there is an incentive effect on the supply of capital, and the increased capital is likely to lead to a further increase in effort because it raises the marginal return to effort. It is still true however that $\alpha<1$.[1]

## 4. *General Remarks on Incentive Schemes*

In the previous section we analysed equilibrium in an economy with a particular incentive scheme, one in which there was no supervision at all of the individual. Such incentive schemes are particularly important in economies (processes) subject to great uncertainty: on the one hand, the uncertainty makes it difficult to ascertain whether a low output was a result of a low level of input by the worker or of " bad luck " on his particular plot of land (see Stiglitz [15]); moreover, the multiplicity of contingencies would entail an impossible degree of complexity in the work contract were it to attempt to specify the action to be taken in each contingency. On the other hand, incentive schemes force the worker to bear a greater share of the risk than he otherwise would.

[1] It should be emphasized that throughout our analysis we have been exploring the implications of first order conditions only. The problem described by (3.17) is not, however, in general a " nice concave " problem, so there may be local optima as well.

Most industrial processes—and many agricultural processes as well—rely on a fair amount of supervision in addition to, or as an alternative to, the incentive schemes sketched here. This is partly because, in production processes involving many individuals working together it may be difficult to separate out each individual's contribution but also partly because the same effort may be obtained without forcing the worker to bear as much risk.

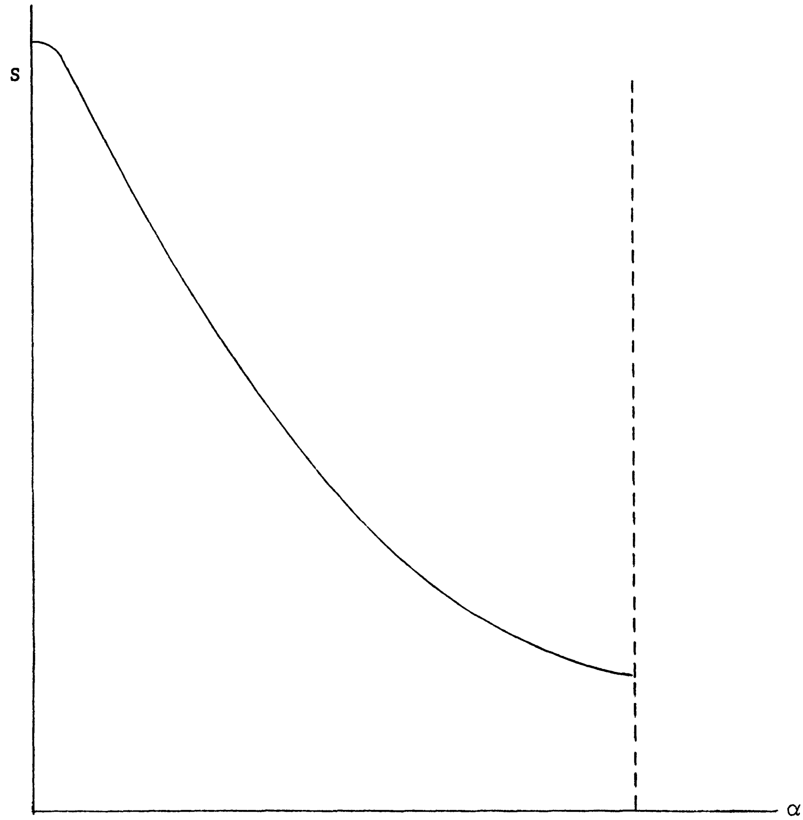


FIGURE 4.1

Supervision would affect the analysis of the previous section in several ways. (*a*) There is probably an important element of fixed costs in supervision. That is, in the rental system, no supervision is required, but if $\alpha < 1$, some supervision is required simply to ensure that the worker delivers the appropriate share of output. At $\alpha = 0$, the landlord replaces incentive structures with direct supervision. If supervision costs look as in Figure (4.1) there is some inducement to use one of the " polar contracts "—a pure rent or pure wage system.[1] (*b*) The more supervision, the less effort (decision making) is required of the individual worker.

In most industrial processes, supervision serves an additional function: a higher degree of supervision results in a more accurate assessment of the individual's efforts; that is, supervision for the purpose of imposing rewards and penalities may be looked at as essentially a sampling of how the individual behaves under various circumstances; the larger the sample, the smaller the variance in the estimate of the worker's effort (the

[1] The level of expenditure on supervision is in fact an endogenous variable, as we suggested earlier. To completely model the consequences of supervision would take us beyond the scope of this paper. Needless to say, the remarks in the last paragraph are meant only to be suggestive.

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" quality " of his performance). Thus, considerations of equity require that a reward-penalty structure system with very high rewards and penalties have a higher degree of supervision than those with low rewards and penalties; employers, of course, are not likely to be directly concerned with equity; but the variability in income from the more random application of the penalty structure requires either a higher share or a higher fixed payment to induce the individual worker to accept a contract.

Many reward-penalty structures take the form of a fixed wage with a reward or penalty for a deviation from the " norm " or average performance. The deviations from the norm are more likely to be detected—and hence the incentives likely to be more effective—under close supervision.

A similar reward-penalty structure could be used in agriculture as an alternative to sharecropping. For instance, the following is a linear reward structure:

$$Y_w = \bar{Y}_w + \rho\left(\frac{f}{l} - \frac{\bar{f}}{l}\right)g,$$

where $\bar{f}/l$ is the average output per man. Since the average pay is just $\bar{Y}_w$, a risk neutral landlord will hire workers up to the point where

$$ef'(el) = \bar{Y}_w$$

(the bars denote averages taken over the population working for a given landlord), and workers supply effort up to the point where

$$\rho f'EU'g + V' = 0.$$

Increasing $\rho$ has several effects. (*a*) If the level of effort of all other individuals were to remain constant, then it would increase the work effort of those near and below the mean but might decrease the work effort of those above the mean.[1] (*b*) An increase in average work effort results in all individuals working still harder; a decrease, in them all working still less hard. (*c*) If there is an increase in average work effort, it decreases the marginal productivity of the average worker if the elasticity of substitution is very low; thus " base pay " is reduced and the income effect again results in their working harder.[2] (*d*) if the base pay is fixed in the short run for some reason, it results in a decrease in the marginal product of the average worker and so workers will be " laid off ".

This simple example illuminates a number of the statements one hears about incentive schemes (or about redistributive schemes more generally): Those who favour a high " $\rho$ " argue that it is only " fair " that those who work harder—the others are, at least partially, just free riders—should get more pay, and point to the favourable effects on work effort of high " $\rho$ ". Those who oppose point out three things: (*a*) incentive schemes result in greater inequality; (*b*) they result in an important externality, commonly known as the rat race: the fact that others work harder " forces " all of them to work harder (i.e. an increase in mean work effort leads each utility maximizing individual to work harder). This is often couched in normative terms: observing the fact that there is an externality, it is alleged that individuals are lead to work *too hard.* I find it too difficult to evaluate such statements, since the normative reference allocation scheme against which this is being compared is not clear.[3] (*c*) Finally, observing the general equilibrium effects of either reduced wages per unit of effective effort or reduced employment, they point out that such schemes may not be in the interests of workers in general.

This raises the general problem of devising reward-penalty-supervisory structures which are " optimal ", a question which we hope to pursue elsewhere.

[1] I.e. the ambiguity arises because of the income effect.
[2] If the elasticity of substitution is not " very low ", the opposite occurs.
[3] See above, p. 245.

### 5. *Concluding Comments*

We began the analysis of this paper with two objectives in mind. (*a*) Could we " explain " the change in the agricultural sector away from the use of sharecropping and to the use of the other two polar contracts: the wage system, primarily in the context of large farms and plantations, and the rental system (leasing-in land), primarily in smaller, family farms. (*b*) Could a model of sharecropping in agriculture shed any light on " shareholding " in the industrial sector.

It initially appeared to me curious that while earlier economies used sharing arrangements in the agricultural sector but not in the urban (artisan) sector, modern economies use sharing arrangements in the manufacturing sector, not in the agricultural sector. As I shall suggest in a moment, the two phenomena may be more closely related than at first seems to be the case.

In these concluding comments, I shall sketch, in a suggestive rather than definitive manner, the implications of our earlier analysis for these problems.

#### *The End of Sharecropping*

First, it appears from the results of Section 2, that the increase in wealth or a change in risk in agriculture cannot in themselves explain the change in the payments system; although they might have had an effect on the actual values of the parameters of the contract, there is no reason to believe that the differences in relative risk aversion between landlords and workers would have increased as a result of an increase in wealth to a sufficient extent to lead the economy near to one of the polar contracts, and even then it could only explain the use of a wage system.[1] And even as the risk goes to zero, the share goes to a limiting value that is not one of the polar (rental or wage) cases.

Secondly, the sharecropping system—in the absence of other methods of risk sharing—is not an inefficient system. It is not as if landlords and workers, anticipating the analysis of Marshall and other economists, discovered that it provided too little incentive to work and therefore they replaced an inefficient payments system with a more efficient one. Indeed, we argued that the labour supply in our simple model is identical to that which would result from a centrally controlled agricultural economy.

The sharecropping system may result in the economy not being *productively efficient*, i.e. the effective labour/land ratio may differ on different plots of land. But this does not demonstrate the inefficiency of the system as a whole given the relevant economic constraints. The sharecropping system is adopted because of its incentive effects (when direct supervision is costly or ineffective) and because of its risk-sharing features. Although the rental system has greater incentive effects, it forces the worker to bear all the risks, and although the wage system allows the landlord, if he is risk neutral, to absorb all the risk, it may force heavy supervision costs on him.

We come then to the following hypotheses concerning the elimination of the sharecropping system.

(*a*) The development of capital markets in which landlords and workers (to the extent that they had capital), could diversify their portfolio meant that the relative importance of sharecropping as a risk sharing arrangement declined.

(*b*) The increasing capital intensity of agriculture meant that either the landlord had to provide strong incentives (the rental system) or had to provide close supervision leading to the wage system. Since there is a natural nonconvexity associated with supervision, it was the larger farms which used the wage system. Similarly, since the rental system required the worker to provide the capital, it was the wealthier workers who became renters, the poorer workers becoming landless workers.

[1] Assuming that landlords are less risk averse than workers. Note that if there is decreasing relative risk aversion, this is what we would expect, since landlords have, in general, a larger income, than workers.

(c) The increase in the rate of technological change has much the same effect as the increased capital intensity of production. The landlord wants the best techniques to be used. Either he must provide a strong incentive to the worker to acquire these techniques or he must supervise the workers closely. Again technological change (information gathering) introduces another non-convexity into the production process (i.e. the fixed costs of acquiring the information may be large relative to the variable cost of disseminating the information among the workers.) This would imply that large farms might be technologically more advanced than smaller farms.[1]

*The Corporation as a Form of Sharecropping*

The relationship between sharecropping and joint stock companies may best be seen in terms of the early formation of such firms. There are two factors of production, " capital " and " entrepreneurship ". The entrepreneur " hires " the capital (or the " capitalists " hire the entrepreneur) to work for him. The capital gets paid a fixed fee (bonds) and/or " a share "; the entrepreneur usually gets a share of the capital value of the firm as his payment (in addition often to a fixed fee). In a " closely held firm ", we have then a simple theory of the optimal (or equilibrium) debt-equity ratio. It is a simple translation of our theory of the contract in agriculture.

There are, however, several important differences between the modern corporation and the simple agricultural economy. The first two differences arise out of the fact that shares of the firm are publicly marketed. In the simpler closely held corporation, there was no ambiguity about the objective of the firm. It was to maximize the utility of the worker (entrepreneur) (and the contract arrangements were such that it simultaneously maximized the utility of the landlord (the capitalist)). Now, we must consider the possibility of a new objective: maximizing the stockmarket value of the firm. Under certain very restricted conditions, again there is no conflict between this objective and maximizing

[1] This model suggests in addition a possible interpretation to the enclosure movement. The essential question is, why did the landlords decide to capture rents from land which they previously had not been capturing? Two hypotheses have been put forward.

The first argues that until that time, the implicit rents were small, but increased land scarcity and technological advances made it more worthwhile for the landlord to capture these rents.

The second argues that there was an institutional change: the development of the market economy and the associated ideas of private ownership led landlords to take for themselves what until then had been viewed as public land.

In our view, the common land may be viewed as part of a complicated payments system. The rights to use the " common " land were part of this payments system, but in some sense all the land belonged to the landlord. Changes in technology and markets made it advantageous to change the payments system, including the " rights " to the common land. For instance, consider a situation where on some of the land crops were grown, providing the basis for survival. The inherent uncertainties in small-scale agriculture and the absence of appropriate insurance markets made a sharecropping system advantageous. For traditional crops grown by traditional methods, variations in effort may not have been significant, and so close supervision was not required. On the other hand, for other commodities, in particular for animal raising (which by its very nature is " capital intensive " (including cattle as " capital ")) incentive effects were important. Thus the landlord gave the right to use the " common land " to the workers; this reduced the share and the fixed fee that workers otherwise would have had to have been paid. There was a potential cost to this particular method: since marginal cost pricing was not used in the allocation of the land, there was always the possibility of overgrazing. Whether or not this was a real cost depended on (a) the implicit terms of the contract, i.e. were the farmers allowed to graze as many cattle as cows or sheep as they wished, or was there any implicit understanding of the maximum " fair " number any single worker could have and (b) whether other constraints (the availability of capital with which to purchase cows or sheep) limited the amount of grazing.

The technological changes to which we referred earlier made it desirable for the landlord to change both the payments scheme and the land allocation among different uses. At the same time, if there were not implicit limits on the use of the common land, increased grazing reduced the value of the use of the common land as a means of payment.

This example is only meant to illustrate how incentives and risk sharing may lead to much more complicated contractual patterns than the simple ones discussed in the text. Whether our possible interpretation of the enclosure movement has any general validity depends on much more detailed empirical evidence.

the utility of the entrepreneur, and of the capitalists. But in general, in the absence of perfect insurance (futures) markets the objectives may not be congruent.[1]

Secondly, again because shares are publicly marketed, the entrepreneur can divest himself of his shares, and this may eliminate one of the major advantages of the sharing arrangements—its incentive effects. This in turn has two effects (*a*) it becomes necessary to devise more complicated incentive arrangements; and (*b*) because the entrepreneur has " inside information ", he can take advantage of " bad news " as well as " good news " to make a profit. It is possible—at least in the short-run—for there to be perverse incentive effects.

Thirdly, because of the greater complexity of the operations of the modern corporation, it becomes much more difficult for the " capitalist " to supervise his worker (" the entrepreneur ") to ascertain even whether he is doing a good job. This becomes all the more important because of the two difficulties we already noted: (*a*) the presence of conflicts of interest and (*b*) the difficulties in providing correct incentives.

Thus, it would appear that the main contribution of the model of risk sharing and incentives in agriculture may be more in extending our understanding of the operations of the closely held firm and the differences between it and the modern widely held corporation, than in its direct implications for the latter.

## APPENDIX

*Proof of Proposition* 4

(1.20*b*) requires

$$\frac{E\left(\frac{\alpha f}{l}g+\beta\right)^{-\eta_w-1}g}{E\left(\frac{\alpha f}{l}g+\beta\right)^{-\eta_w-1}}=\frac{E\left(\frac{(1-\alpha)f}{l}g-\beta\right)^{-\eta_r-1}g}{E\left(\frac{(1-\alpha)f}{l}g-\beta\right)^{-\eta_r-1}}.$$

Assume $\eta_w>\eta_r$. Assume $\beta\leqq 0$. Then

$$\frac{E\left(\frac{\alpha f}{l}g+\beta\right)^{-\eta_w-1}g}{E\left(\frac{\alpha f}{l}g+\beta\right)^{-\eta_w-1}}\leqq\frac{Eg^{-\eta_w}}{Eg^{-\eta_w-1}}<\frac{Eg^{-\eta_r}}{Eg^{-\eta_r-1}}\leqq\frac{E((1-\alpha)fg-\beta l)^{-\eta_r-1}g}{E((1-\alpha)fg-\beta l)^{-\eta_r-1}}.$$

To see the first inequality, write

$$\tilde{\phi}\equiv\frac{\left(\frac{\alpha f}{l}g+\beta\right)^{-(\eta_w+1)}}{E\left(\frac{\alpha f}{l}g+\beta\right)^{-(\eta_w+1)}}$$

$E\tilde{\phi}=1$ for all $\beta$, i.e. $E\tilde{\phi}_\beta\equiv 0$

$$\tilde{\phi}_\beta=\frac{-\left(\frac{\alpha f}{l}g+\beta\right)^{-(\eta_w+2)}(\eta_w+1)E\left(\frac{\alpha f}{l}g+\beta\right)^{-(\eta_w+1)}+\left(\frac{\alpha f}{l}g+\beta\right)^{-(\eta_w+1)}E\left(\frac{\alpha f}{l}g+\beta\right)^{-(\eta_w+2)}(\eta_w+1)}{\left[E\left(\frac{\alpha f}{l}g+\beta\right)^{-(\eta_w+1)}\right]^2}$$

$$\gtrless 0\ \text{as}\ \frac{\alpha f}{l}g+\beta\gtrless\frac{E\left(\frac{\alpha f}{l}g+\beta\right)^{-(\eta_w+1)}}{E\left(\frac{\alpha f}{l}g+\beta\right)^{-(\eta_w+2)}}.$$

[1] See, for instance, Stiglitz [14].

Thus, there exists some $\hat{\theta}$, $g(\hat{\theta})$ such that

$$\tilde{\phi}_\beta \gtrless 0 \text{ as } g(\theta) \gtrless g(\hat{\theta}).$$

Hence

$$\frac{\partial E\tilde{\phi}g}{\partial \beta} = E\phi_\beta g = E\phi_\beta(g-\hat{g})>0.$$

The third inequality follows in exactly the same way. To see the second inequality, write

$$\tilde{\psi} \equiv \frac{g^{-z-1}}{Eg^{-z-1}}$$

$$E\tilde{\psi} = 1$$

$$E\tilde{\psi}_z = 0$$

$$\tilde{\psi}_z = \frac{-g^{-z-1}\ln gEg^{-z-1}+g^{-z-1}Eg^{-z-1}\ln g}{(Eg^{-z-1})^2}$$

$$\gtrless 0 \text{ as } \ln g \lessgtr \frac{Eg^{-z-1}\ln g}{Eg^{-z-1}} \equiv \ln \hat{g}$$

$$\frac{\partial E\tilde{\psi}g}{\partial z} = E\tilde{\psi}_z g = E\tilde{\psi}_z(g-\hat{g})<0.$$

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